Stability of Three-Dimensional Boundary-Layers

AE 549 Linear Stability Theory and Laminar-Turbulent Transition

Prof. Dr. Serkan ÖZGEN

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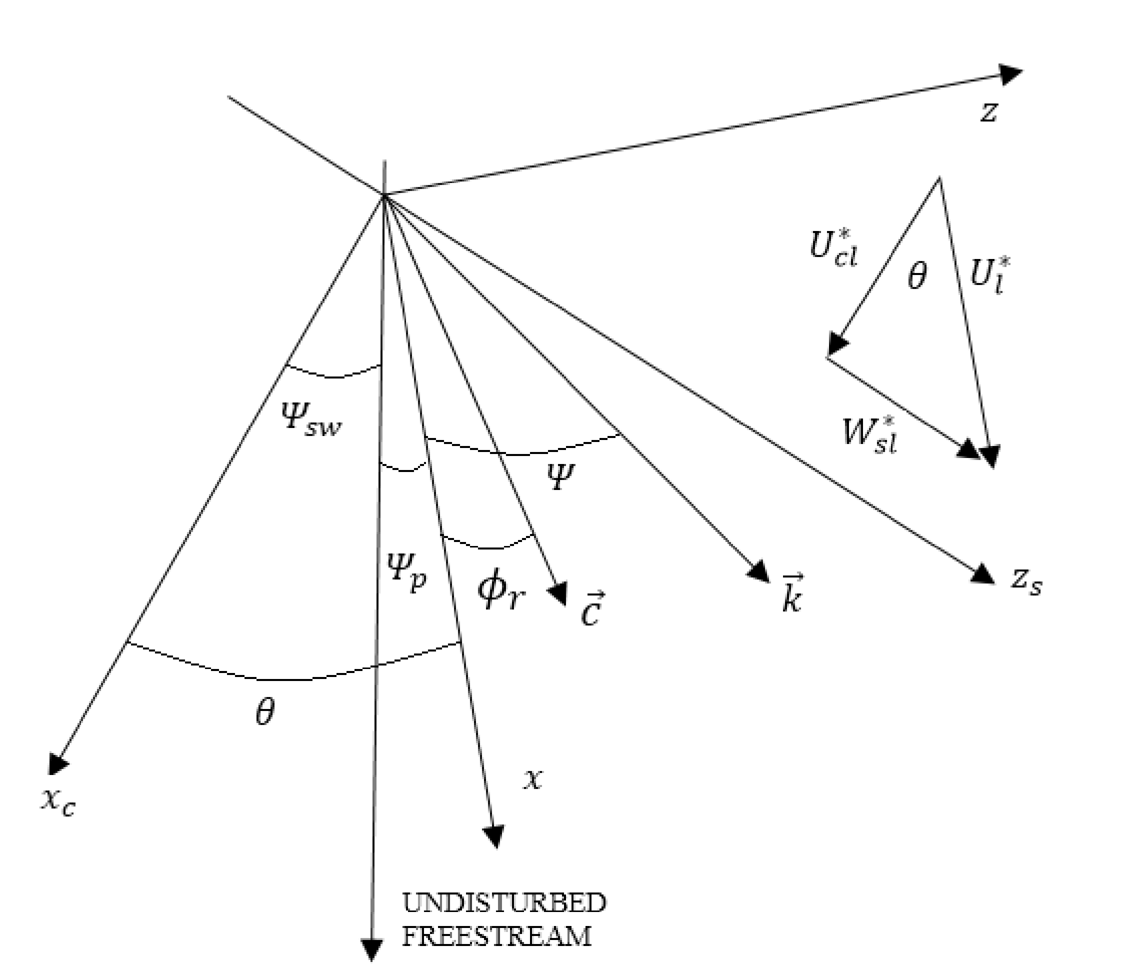
Three-dimensional stability

- A fundamental difference between the stability of 3-D and 2-D boundary-layers is that a 3-D boundary-layers is subject to **crossflow instability**.
- To understand the effect of three-dimensioanlity of the mean flow on stability, it is necessary to have a **family of boundary-layers**, where the magnitude of the crossflow can be varied in a systematic manner.
- The two-parameter yawed-edge (swept leading edge) flows are suitable for this purpose resulting in **Falkner-Skan-Cooke** family of profiles.

Flow geometry for three-dimensional b.l. flow



3-D flow with pressure gradient

- The two parameters are:
  - Falkner-Skan or Hartree parameter β_H,
  - Flow angle θ, which is the ratio of the spanwise freestream velocity to the chordwise freestream velocity, $\tan\theta = W^*_{sl}/U^*_{cl}$.
- The inviscid velocity in the plane of the wedge and normal to the leading edge (in x_c-direction, i.e. chordwise velocity) is defined as:

  $$U^*_{ce} = C^* x_c^{*m},$$

  where the wedge angle is $\beta\pi/2$ and $\beta = 2m/m + 1$, with m being the dimensionless pressure gradient defined as:

$$m = \frac{x^*_c}{U^*_{ce}} \frac{dU^*_{ce}}{dx^*_c}$$

- The velocity parallel to the leading edge (in z_s direction, i.e. spanwise velocity) is

  $$W^*_{se} = constant.$$

3-D flow with pressure gradient

- Flow in the chordwise direction, x_c is defined by the Falkner-Skan equation, which is independent of the spanwise flow, and $f' = U_c$:

  $$2f''' + ff' + \beta_H(1 - f'^2) = 0.$$

- Falkner-Skan length scale is defined as:

  $$L^* = \left[\frac{v^* x_c^*}{(m+1)U_{ce}^*}\right]^{1/2}.$$

- Once f is solved, flow in the spanwise direction, z_s is defined from the following equation:

  $$g'' + fg' = 0,$$

  where $g = W_s^*/W_{se}^*$.

- Boundary conditions:

  $f'(0) = g(0) = 0,$ (no slip)

  $f'(y) \to 1, g(y) \to 1$ as $y \to \infty.$ (freestream)

.

3-D flow with pressure gradient

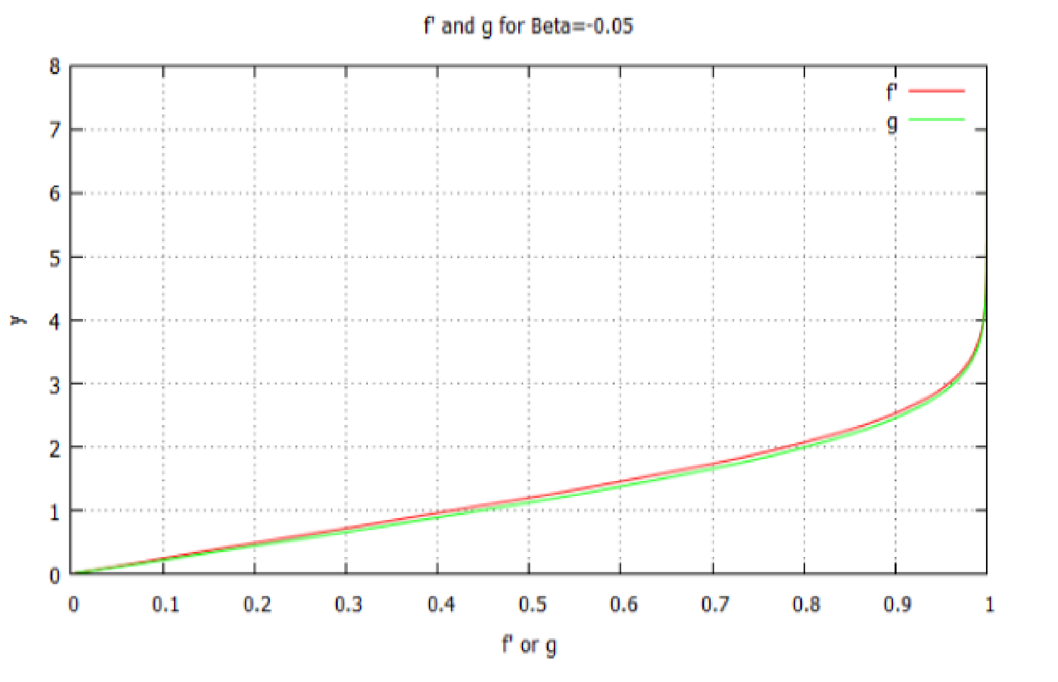
- Dimensionless streamwise (x-direction) and crossflow (z-direction) velocity components:

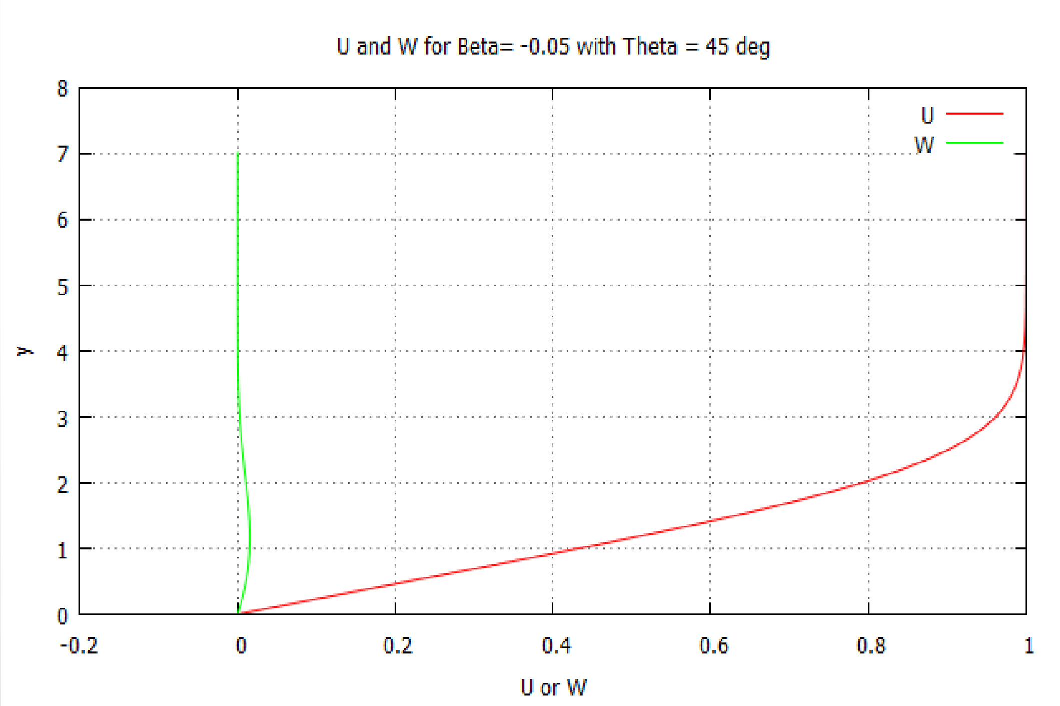
  $$U(y) = f'(y)\cos^2\theta + g(y)\sin^2\theta,$$

  $$W(y) = [-f'(y) + g(y)]\cos\theta\sin\theta.$$

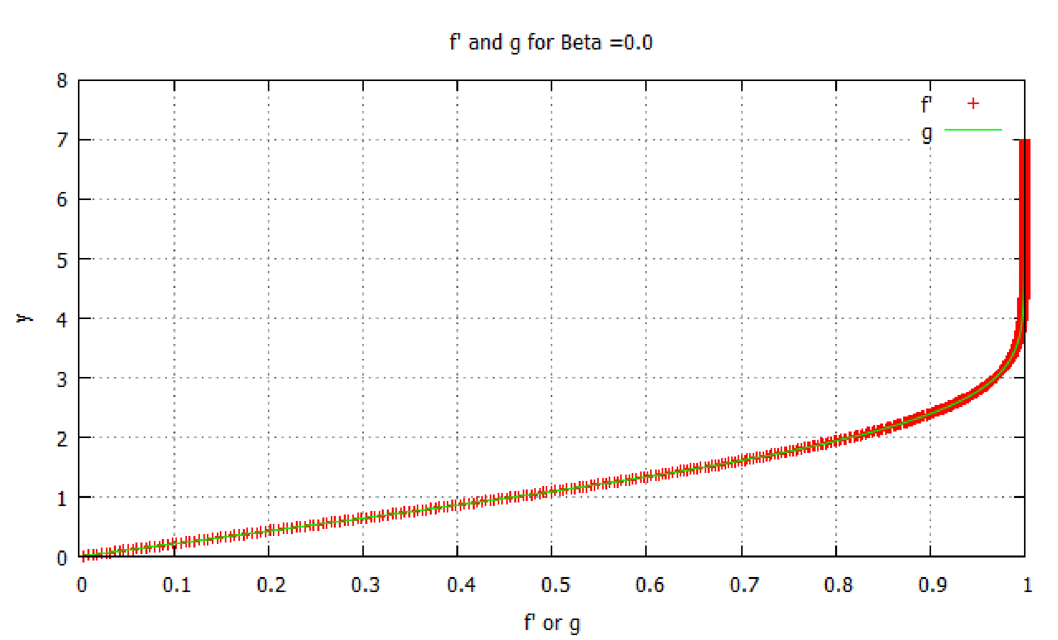
- Falkner-Skan parameter β fixes both $f'(y)$ and $g(y)$.
- It can be seen from the above equation that all crossflow profiles $W(y)$ have the same shape for a given pressure gradient, i.e. β value.
- Magnitude of the crossflow velocity will change with flow direction θ.
- However, streamwise profiles $U(y)$ will change shape as θ varies.

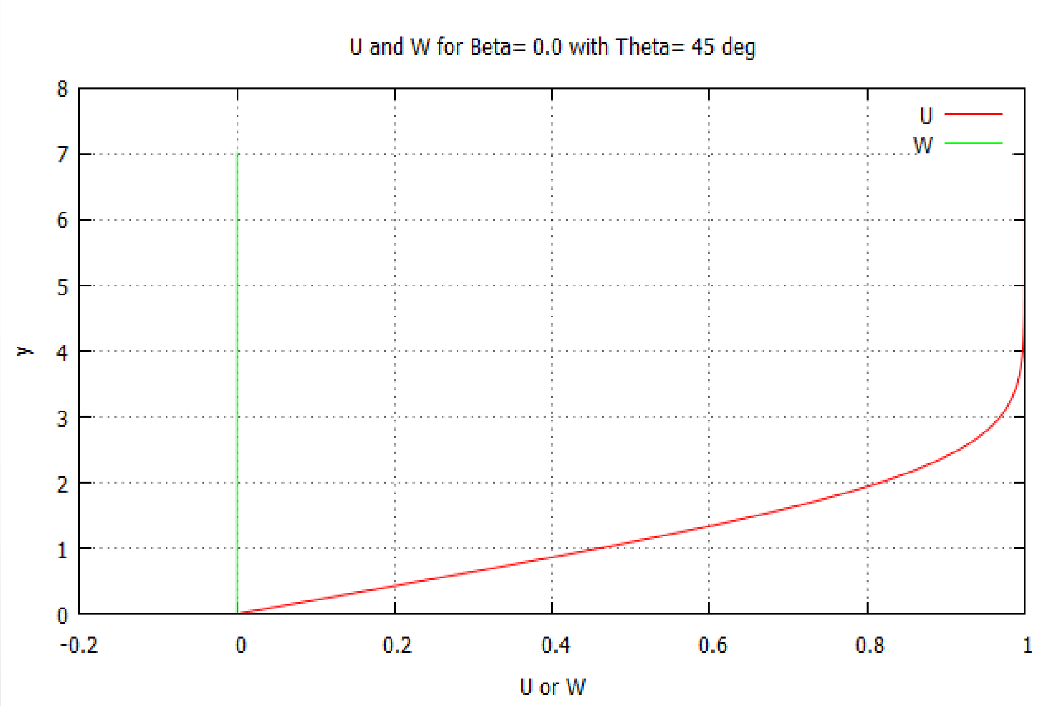
Velocity profiles for $\beta_H = -0.05$ and $\theta = 45°$



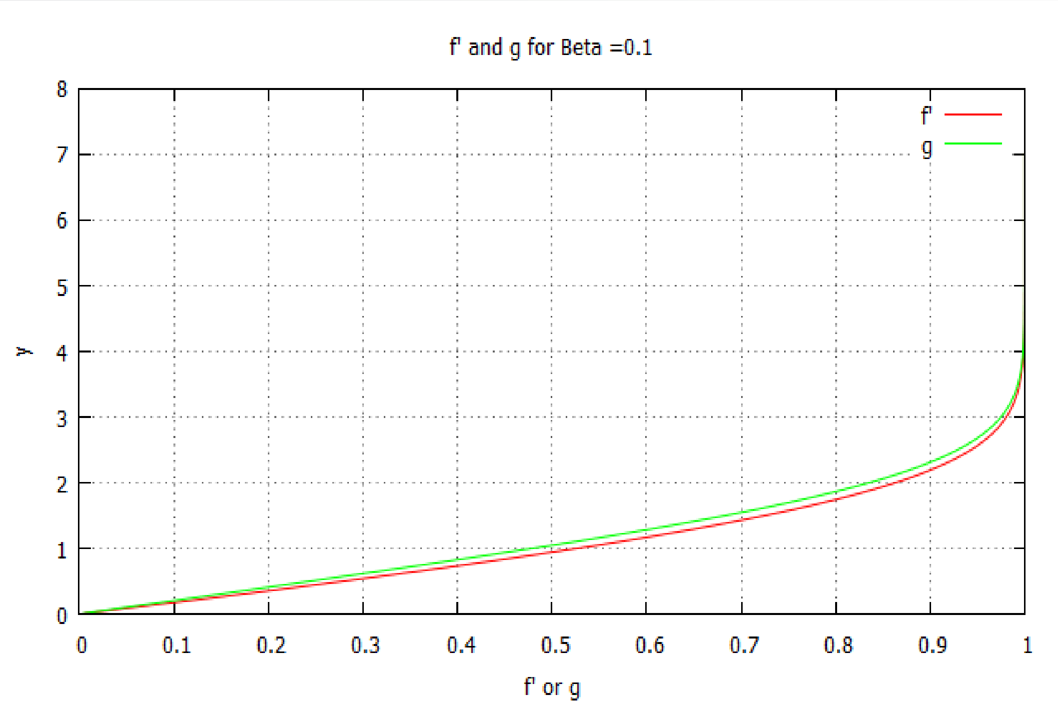


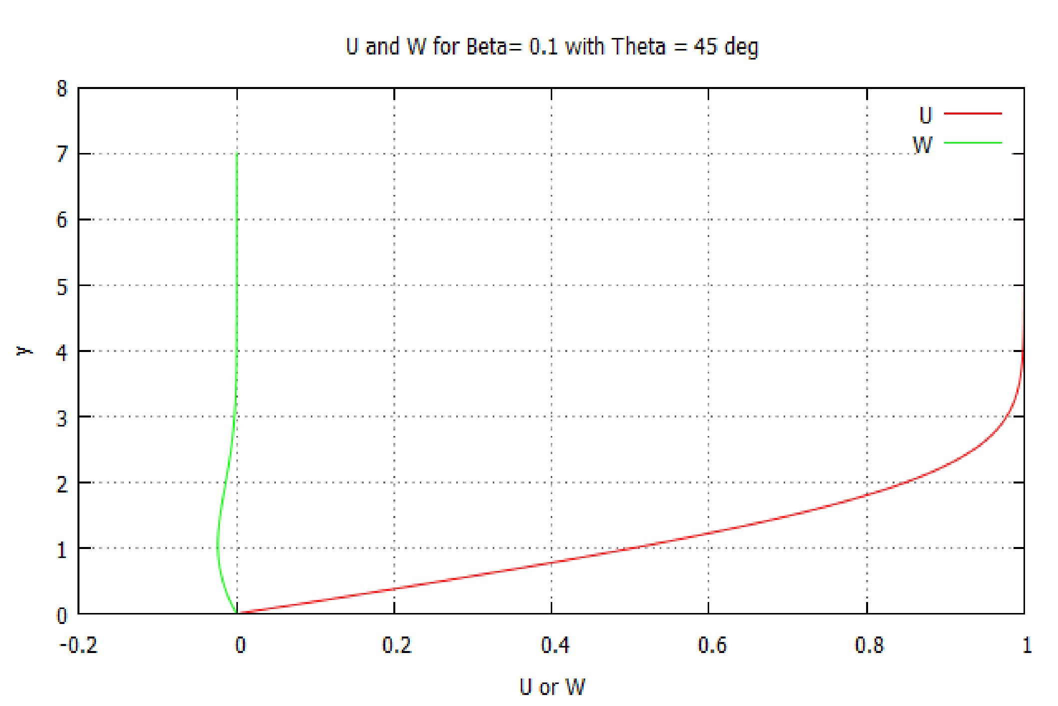
Velocity profiles for $\beta_H = 0$ and $\theta = 45°$



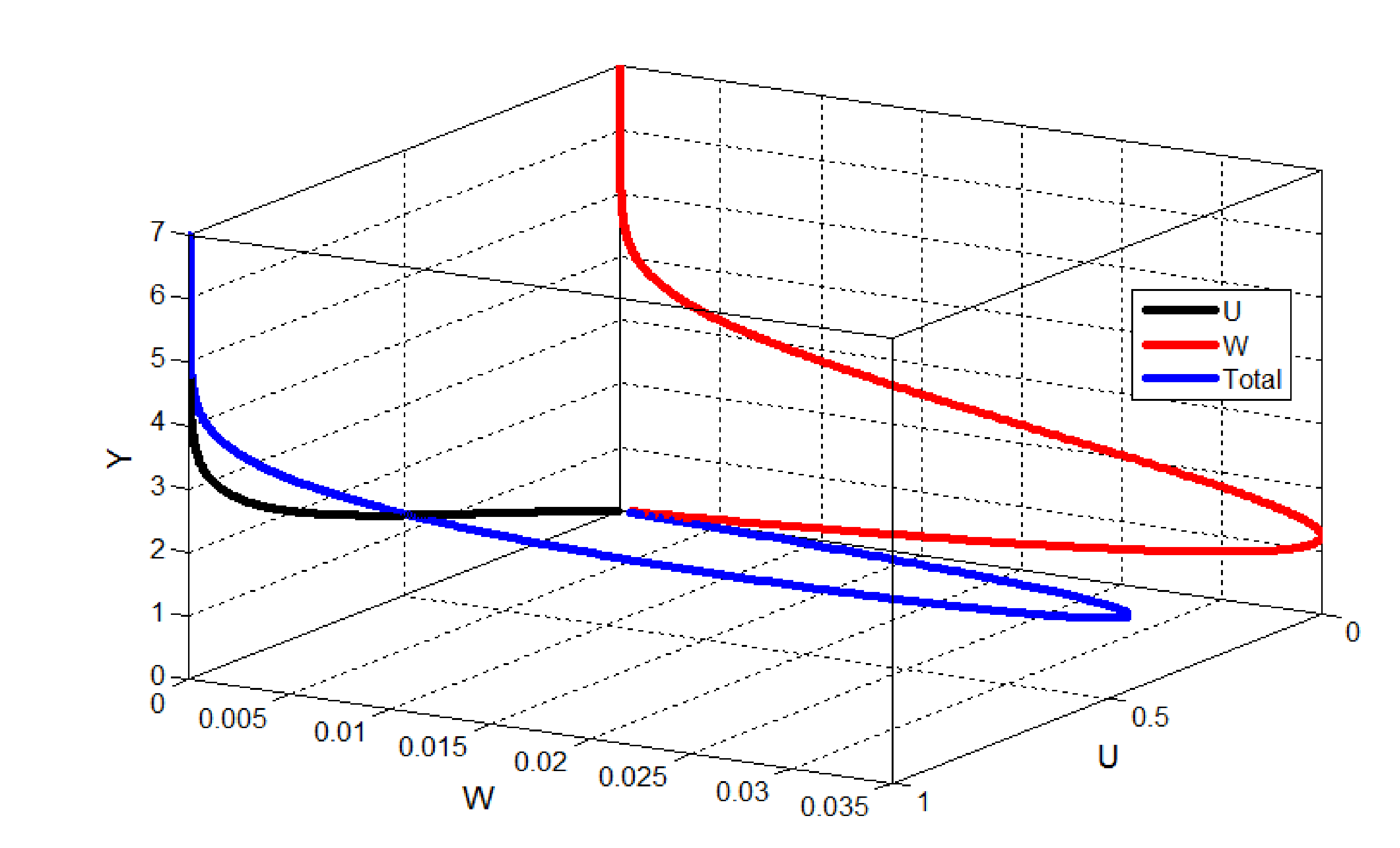


Velocity profiles for $\beta_H = 0.1$ and $\theta = 45°$

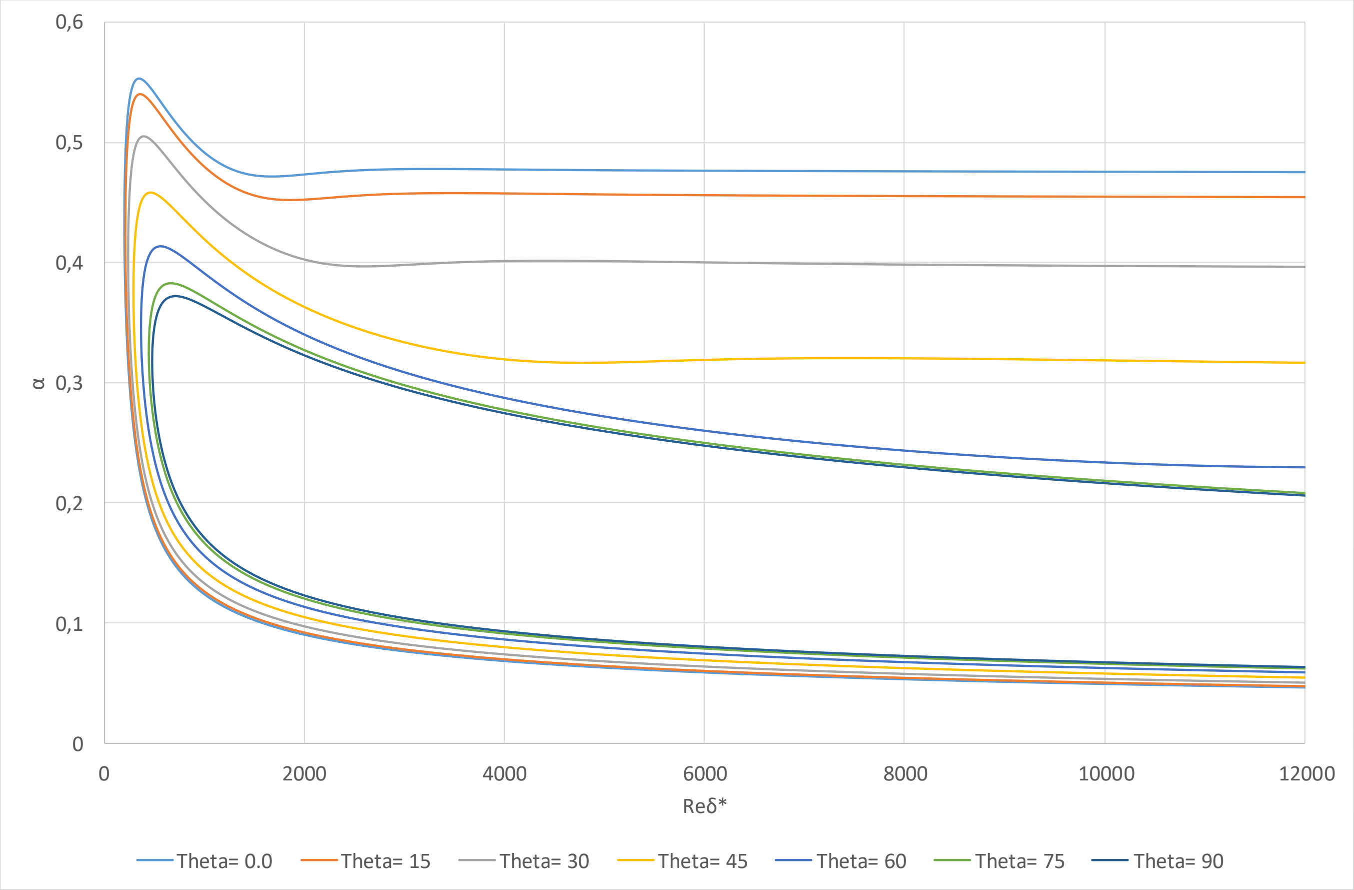




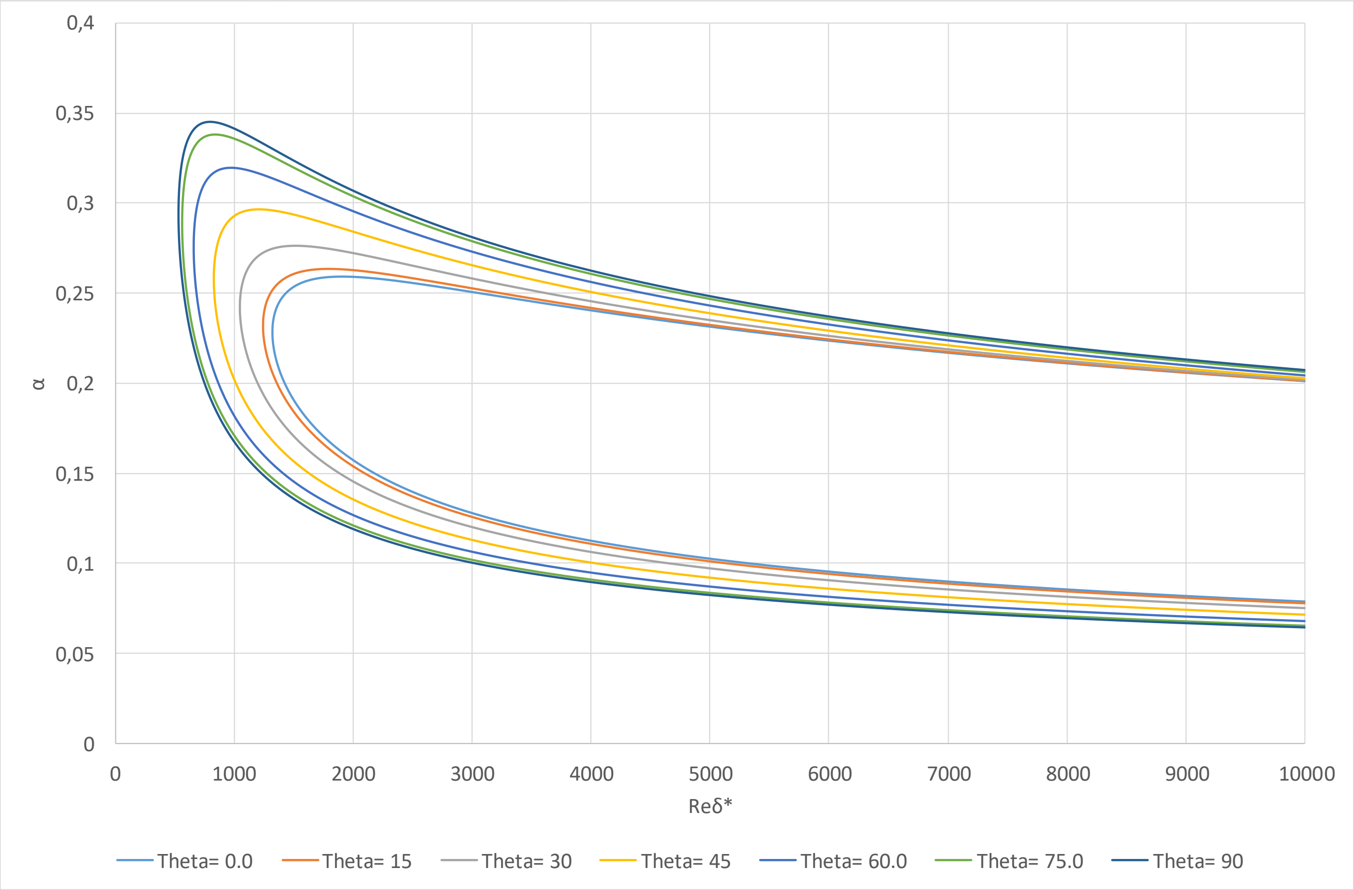
Composite profile for $\beta_H = -0.1$ and $\theta = 45°$



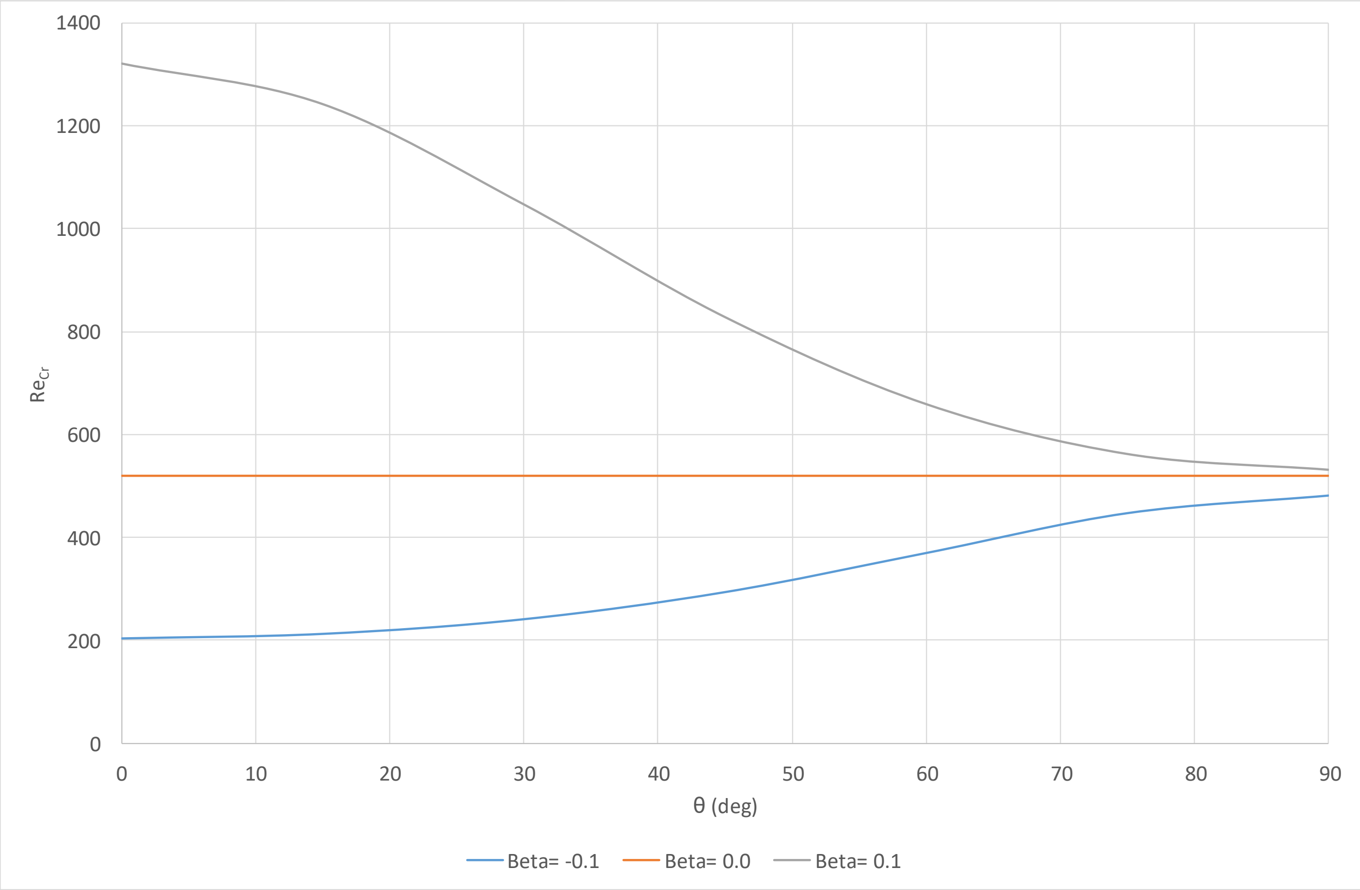
Neutral stability curves for $\beta_H = -0.1$



Neutral stability curves for $\beta_H = 0.1$



Effect of flow angle on Re_{cr} for $\beta_H = -0.1, 0, 0.1$



Conclusions

- For $\beta<0$, increasing flow angle renders the flow more stable.
- For $\beta>0$, increasing flow angle renders the flow more unstable.
- Flow angle has no effect on stability when $\beta = 0$.
- When the flow angle is around 0^o, the critical Reynolds number of the three-dimensional flow is very close to that of the two-dimensional flow.

  When $\theta = 0^o, U(y) = f'$ and $W(y) = 0$.
- When the flow angle is $\theta=90°$, the critical Reynolds number of the three-dimensional flow is very close to that of the Blasius flow($\beta = 0°$).

  When $\theta = 90^o$, $U(y) = g$, which is very close to the Blasius profile and $W(y) = 0$.