recorded FBG sensor strain responses, were able to identify the position of the debond in the damaged beam specimen. Further work is being conducted to correlate features in the singularity spectrum with damage severity in FRP sandwich composite beams.

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Low Level Damage Characterisation in FRP Sandwich Beams using the Lipschitz Exponent

T. A. DAWOOD, R. A. SHENOI, S. M. VERES and M. SAHIN

ABSTRACT

A wide variety of non-destructive damage detection methods are available for damage assessment of composite structures, which are used extensively in aerospace and marine applications. The work presented here deals with the development of a new Lipschitz exponent based technique, estimated using a complex Gaussian wavelet, to characterise very small levels of localised damage in cantilevered beams. The estimated Lipschitz exponent is presented as a function of position along the beam to simultaneously locate and assess the severity of damage in the beam. Using the fundamental mode shape, the method was initially verified for aluminium beams and then successfully adopted to identify defects in foam core fibre reinforced plastic sandwich beams.

INTRODUCTION

Fibre reinforced plastic (FRP) sandwich structures are widely used in a variety of applications. However, they still remain vulnerable to internal defects which can occur during the manufacturing process or during its service life. Since it is not practical to take the structure apart to identify these internal defects, non-destructive damage evaluation (NDE) methods are required [6][16]. These methods however are not very practical but, can be overcome by using an approach based on measuring structural vibration characteristics. These characteristics are mainly based on modal data such as natural frequencies and mode shapes [1][15].

Lately attention has been focused on using a relatively new technique known as wavelet analysis to examine structural vibration responses for signs of structural damage due to its unique zooming ability or localization property [7][8]. However, most of the wavelet based approaches investigated so far have failed to come up with an approach that utilises a single parameter to identify damage from structural defection profiles. Hong addressed this issue by applying a wavelet transform modulus maxima method (WTMM) to the fundamental displacement mode shape of a beam, to characterise a defect, by estimating the Lipschitz exponent from the locus of the modulus maxima [5]. We also verified Hong's method for cantilevered sandwich composite beams [2]. In addition Peng and Robertson used the Lipschitz exponent as a damage sensitive feature to identify damage in rotating shafts and transients in earthquake signals respectively [12][14].

Fluid-Structure Interactions Research Group; School of Engineering Sciences; University of Southampton.

Considering the approach by Hong, it is important to highlight the fact that two mutually exclusive methods were used to locate and quantify damage in a beam [5]. Recently, we proposed to use the Lipschitz exponent directly as a function of spatial distance along the beam to simultaneously locate and assess the severity of defect present in a beam, thus bypassing the initial modulus maxima calculation [3]. This approach was applied to numerically simulated fundamental mode shapes of cantilevered aluminium and sandwich composite beams for a number of different damage scenarios. A strong singularity which manifests itself as a dip in the spatially varying Lipschitz exponent was observed at the position of the defect. Also a good correlation was observed between the severity of the defect and the depth of the dip. It is also worth mentioning that the use of a complex second order Gaussian wavelet, used to estimate the Lipschitz exponent from the continuous wavelet transform, highlighted small to medium levels of damage very well.

This paper builds upon the research performed previously and investigates the ability of the spatially varying Lipschitz exponent approach to identify localized stiffness reductions of less than ten percent in metal and sandwich composite cantilevered beam specimens.

WAVELET TRANSFORM

Continuous Wavelet Transform

For any square-integrable signal $f(x)$ the Continuous Wavelet Transform (CWT) is defined as follows [9].

$$Wf(u,s)=\int_{-\infty}^{+\infty} f(x)\psi^{*}_{u,s}(x)dx=\int_{-\infty}^{+\infty} f(x)\frac{1}{\sqrt{s}}\psi^{*}\left(\frac{x-u}{s}\right)dx \tag{1}$$

where $\psi^{*}(x)$ is the conjugate of the mother wavelet $\psi(x)$. The function $\psi_{u,s}(x)$ is defined as a dilation by a scale s and a translation by a step u of the mother wavelet $\psi(x)$. Equation (1) holds as long as $\psi(x)$ satisfies the condition

$$C_{\psi}=\int_{0}^{+\infty}\frac{\left|\hat{\psi}(\omega)\right|^{2}}{|\omega|}d\omega<+\infty \quad \text{where } \hat{\psi}(0)=0 \tag{2}$$

Lipschitz Exponent and Singularity Measurement

The Lipschitz exponent α is often used as a measure of a signals local regularity and is defined by as follows: A function f is pointwise Lipschitz $\alpha\geq 0$ at $x=v$ if there exists $K>0$ and a polynomial p_v of degree m (where m is the largest integer satisfying $m\leq\alpha$) such that from [10].

$$|f(x)-p_{v}(x)|\leq K|x-v|^{\alpha} \tag{3}$$

The CWT can be used to estimate the Lipschitz exponent α by using a mother wavelet with $n>\alpha$ vanishing moments that satisfies the following property

$$\int_{-\infty}^{+\infty} x^{k}\psi(x)dx=0 \quad \text{for } 0\leq k<n \tag{4}$$

This allows the CWT to focus on the singular part of f.

Mallat showed that if $f(x)$ is Lipschitz $\alpha\leq n$ at $x=v$, then the asymptotic behaviour of $Wf(u,s)$ near $x=v$ becomes from [10].

$$|Wf(u,s)|\leq As^{\alpha+(1/2)}\left(1+\left|\frac{u-v}{s}\right|^{\alpha}\right) \tag{5}$$

If u is in the cone of influence of v, Equation (5) reduces to

$$|Wf(u,s)|\leq A's^{\alpha+(1/2)} \tag{6}$$

Therefore, it can be inferred from Equation (6) that, high amplitude wavelet coefficients are located within the cone of influence of the singularity [10].

Modulus Maxima

The Lipschitz exponent α of f at v is dependent on the decay of $|Wf(u,s)|$ which is controlled by its local modulus maxima values [9]. Chaining the local modulus maxima points across scales to create a maxima line enables the identification of singularities, as their presence causes the locus of the maxima line to converge at fine scales to the singularity's position. To ensure that the locus of the modulus maxima line converges at fine scales, the mother wavelet $\psi(x)$ used in the CWT is represented as the nth derivative of a Gaussian function $\theta(x)$ such that $\psi(x)=(-1)^{n}(d^{n}\theta(x)/dx^{n})$. In order to estimate α numerically, a more convenient form of equation (6) is

$$\log_{2}|Wf(u,s)|\leq\log_{2}A+(\alpha+\frac{1}{2})\log_{2}s. \tag{7}$$

It is important to note here that Equation (6) refers to the characterisation of isolated singularities.

DAMAGE DETECTION OF BEAM STRUCTURES

Very Small Damage Characterisation

The presence of a defect in a beam causes a localised change in the profile of its displacement mode shape. This localised change however is quite difficult to identify for all but very large levels of damage. However, it is envisaged that this localised change manifests itself in the form of a singularity in the beam's displacement mode shape.

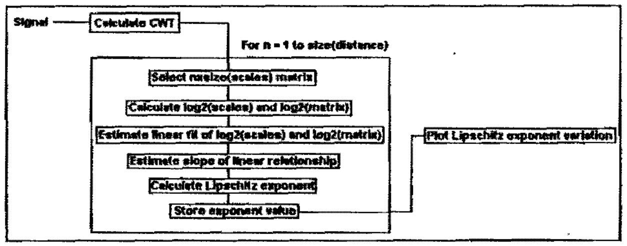

Figure 1: Algorithm used to estimate the Lipschitz exponent for all positions along the beam.

Previously, we developed a novel approach to identify this singularity, which involves estimating the Lipschitz exponent for all positions along the beam, from its fundamental mode shape [3]. The MATLAB algorithm used to implement this approach is shown as a flow chart in figure 1. Using a complex second order Gaussian mother wavelet in the CWT, this algorithm proved effective in identifying localised defects with 20%, 50%, 60%, 90% and 99% stiffness reduction in numerically simulated cantilevered aluminium and FRP sandwich composite beams [3]. In this section we show that by using higher orders of the complex Gaussian wavelet, very small levels of localised stiffness reduction (less than ten percent stiffness reduction) can be characterised using this approach.

Application to Aluminium Beam

The algorithm in figure 1 was initially applied to the fundamental mode shape, obtained through numerical simulation, of undamaged and damaged aluminium beams in a cantilevered configuration. The aluminium beam model used consisted of 145 1D, 1mm long elements, where the stiffness in element 72 was modified to represent a localised defect.

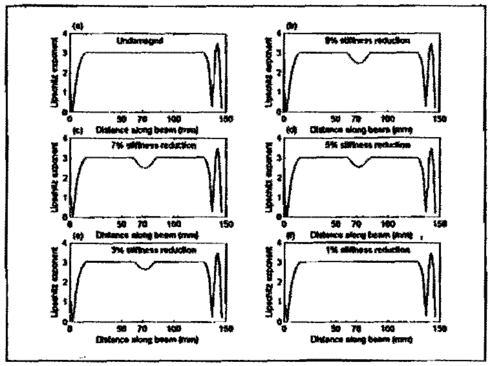

Figure 2: Behaviour of the spatially varying Lipschitz exponent using a complex Gaussian wavelet of order six for an aluminium beam.

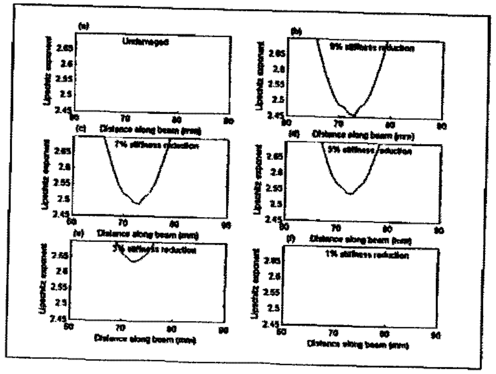

Figure 3: Zoomed view of the spatially varying Lipschitz exponent highlighting the variations in the depth of the dip.

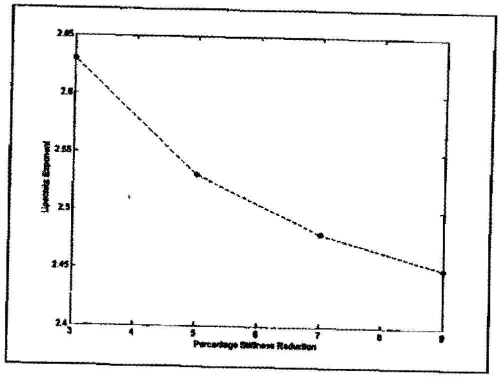

Figure 4: Relationship between Lipschitz exponent and damage extent for the case of an aluminium beam.

The Lipschitz exponent was estimated using the CWT with a complex Gaussian wavelet of order six with scales from 0.55–3.65 in steps of 0.001. The variation of the Lipschitz exponent with the distance along the beam is shown in Figure's 2(a)–2(f) for an undamaged beam and five damaged beams with 1%, 3%, 5%, 7% and 9% stiffness reduction at element 72 respectively. In Figure's 2(a)–2(f), the large variations in the Lipschitz exponent close to the ends of the beam can be attributed to effect of the beam boundary conditions. In the case of the clamped end, the variation isn't as large as the effect seen towards the free end. This is very similar to the observations made by [4].

Compared with the undamaged case shown in figure 2(a), the location of the localised defect is clearly visible by the dip in the spatially varying Lipschitz exponent in figure's 2(b)-2(e). Figure 2(f) however does not indicate the presence of a localised defect as its stiffness reduction of 1% is so small that the beam's fundamental mode shape is identical to that of the undamaged case.

Also, although not visibly apparent, there is a variation in the depth of the dip in figure's 2(b)–2(e). A zoomed view shown in figure's 3(a)–3(e) clearly highlights this observation. A plot of the magnitude of the dip minimum against the defect severity is shown in figure 4. It is clear in this case that there is a strong relationship between the dip minimum and defect severity which correlates quite well to the observations made by [2][3][5].

Application to a FRP Sandwich Composite Beam

Once the approach was verified for an aluminium beam model it was then applied to a FRP sandwich beam model. The ANSYS finite element (FE) simulation tool was used to model a FRP sandwich beam in undamaged and damaged states. The dimensions of the beam model used in the simulation had a length of 360mm, a width of 33mm and a total thickness of 14mm which included a core thickness of 12mm and a skin thickness of 1mm on either side of the core. The skin of the beam model consisted of four equal layers with an orientation of *0°/90°/90°/0°*. In order to extract the fundamental mode shape, a modal analysis was performed on the beam model in a cantilevered configuration.

In order to simulate a defect, such as a debond between the skin and core, an equal percentage reduction in stiffness was introduced to the core and skin 97.5mm away from the clamped end of the beam as shown in figure 5. The size of the defect introduced was 10mm in length. The simulation was performed for an undamaged case and five damage cases with equal reductions in core and skin stiffness of 1%, 3%, 5%, 7% and 9% respectively. The one dimensional displacement mode shape for each case was then obtained.

Using the CWT with a complex Gaussian wavelet of order three with scales from 5–10 in steps of 0.05, the variation of the Lipschitz exponent with distance for the FRP sandwich beam was calculated and is shown in figure's 6(a)–6(f) for an undamaged beam and five damaged beams. It is clear that the behaviour of the spatially varying Lipschitz exponent for the FRP sandwich beam is quite similar to what was observed in the case of the aluminium beam. A plot of the magnitude of the dip minimum against defect severity for the composite beam is shown in figure 7. It is interesting to observe a linear relationship between the dip minimum and defect severity in this case compared with the aluminium beam. However, the general trend compared with the aluminium beam remains the same where the magnitude of the Lipschitz exponent decreases with increasing severity.

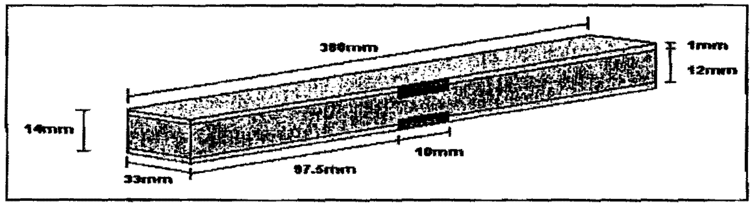

Figure 5: Dimensions of sandwich beam model used in FE analysis.

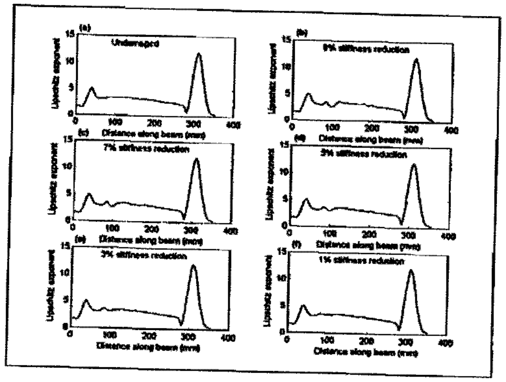

Figure 6: Behaviour of the spatially varying Lipschitz exponent using a complex Gaussian wavelet of order three for a composite beam.

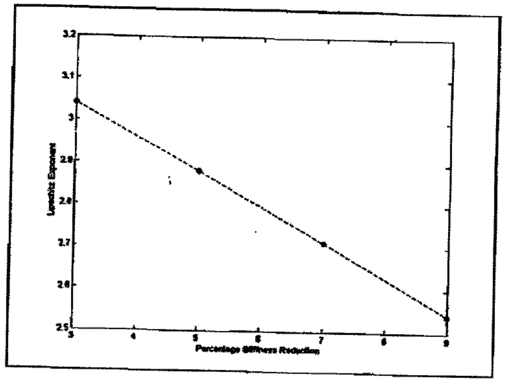

Figure 7: Relationship between Lipschitz exponent and damage extent for the case of a FRP sandwich composite beam.

CONCLUSION

The intention of this investigation has been to ascertain the viability of using the spatially varying Lipschitz exponent based approach, developed previously, to identify very small levels of localised damage in cantilevered FRP sandwich composite beams [3]. Using this approach with higher order complex Gaussian wavelets of order six and order three, localised stiffness reductions of 9%, 7%, 5% and 3% were clearly highlighted in cantilevered aluminium and FRP sandwich composite beams respectively. A strong relationship was also observed between the dip minimum of the Lipschitz exponent, corresponding to the localised defect, and the stiffness reduction.

Although the effects of the boundary conditions limit the ability of this damage detection method to identify localised defects close to the edges of the beam, it makes up for it by having the advantage of being able to use only the damaged mode shape profile to simultaneously identify, localise and assess the severity of the defect.

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A Fuzzy-Aided Wavelet Damage Recognition for Intelligent Structural Health Monitoring

M. M. REDA TAHA, A. NOURELDIN and T. J. ROSS

ABSTRACT

An autonomous IT-based structural health monitoring (SHM) system is developed to overcome limitations by classical SHM systems. The new system denoted ISHM operates in real-time environments. The ISHM system incorporates intelligent algorithms, to observe and extract features from the monitored structure dynamics in the time pattern and to provide pattern recognition of those features subsequently for damage identification.

The proposed ISHM system augments artificial neural networks (ANN) and wavelet multi-resolution analysis (WMRA) to extract features of the dynamics signals, to compute an energy index in the wavelet pattern and to relate this index to the level of structural health using means of fuzzy pattern recognition. The system is examined using data simulated from finite element analysis of a prestressed concrete bridge with- *a priori*- known levels of damage. Features of the new SHM system are discussed.

INTRODUCTION

The strategic and monetary values of the civil infrastructures worldwide necessitates the development of structural health monitoring (SHM) systems that can accurately monitor the structural response due to real-time loading conditions and detect damages in the structure and report location and nature of this damage.

Mahmoud M. Reda Taha and Timothy J. Ross, Department of Civil Engineering, The University of New Mexico, Albuquerque, NM, USA 87131

Abouelmagd Noureldin, Department of Electrical and Computer Engineering, Royal Military College of Canada, Kingston, Ontario, Canada K7K 7B4