

# APPLICATION OF $\mu$ -SYNTHESIS ACTIVE VIBRATION CONTROL TECHNIQUE TO A SMART FIN

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## ABSTRACT

This study presents a  $\mu$ -synthesis active vibration control technique applied to the sinusoidally forced vibrations of a smart fin. The smart fin consists of a cantilever aluminum passive plate-like structure with surface bonded piezoelectric (PZT, Lead-Zirconate-Titanate) patches. The study presents the design of controllers via  $\mu$ -synthesis, which effectively suppress the vibrations of the smart fin due to its first flexural and first torsional modes. Two different experimental set-ups are used in the study. In the first set-up the response is acquired by the strain gages and the vibration suppression is achieved by using Sensortech SS10 controller unit. In the second set-up the response is obtained by using a laser displacement sensor and the vibrations are suppressed through LabVIEW based programs. The effectiveness of the  $\mu$ -synthesis technique in active vibration control of the smart fin is also presented.

## 1. INTRODUCTION

The smart structure is a structure, which can sense the external disturbances and respond to those in real time to maintain the mission requirements. Smart structures consist of passive structures together with highly distributed active devices called smart materials and controller units. The smart materials are either embedded or attached to the passive structure.

The effectiveness of piezoelectric actuators for the vibration suppression of one-dimensional structures was shown by Crawley and de Luis [1]. This application was extended to two-dimensional structures by Dimitridis *et. al.* [2]. Dosch *et. al.* [3] studied on the structural design such as optimal actuator and sensor placement, size and power requirements of the actuators by using finite element methods for the active vibration control applications.

The design of controllers via  $\mu$ -synthesis method for the vibration suppression of flexible structures was studied by Balas *et. al.* [4,5] and Nalbantoğlu [6].

Yaman *et. al.* [7,8,9] and Çalışkan [10] analyzed various smart structures. The fully coupled structural models of a smart beam, a smart rectangular plate and fin shaped plate called smart fin were obtained from ANSYS® (v5.6) and for the control purposes, the system models were identified from the relevant experimental data. Based on those models,  $H_\infty$  controllers, which effectively suppressed the free, in-vacuo vibrations of those smart structures due to their first two flexural modes, were designed. The designed controllers were also implemented for the smart beam [11]. In those studies, the suitability of the  $H_\infty$  design technique in the modeling of uncertainties and the evaluation of the robust performance of the system were also demonstrated.

This study presents the design of controllers via  $\mu$ -synthesis, which effectively suppress the vibrations of the smart fin due to its first flexural and first torsional modes.

## 2. THE SMART FIN

The smart fin was constructed by symmetrically attaching twenty-four PZT patches (25mm x 25mm x 0.5mm, Sensortech BM500 type) as actuators and six strain gages (OMEGA-SG-7/350-LY13) as sensors on a passive aluminum plate-like structure called the smart fin. In the analysis, the smart fin was considered as being in clamped-free configuration.

The actuators and sensors were placed on the determined locations having high strain by using the finite element analysis [10]. Although in the structural modeling, PZT's on both side of the smart fin were used; during the real time implementations, the piezoelectric actuators of only one side were utilized. This inevitably halved the desired actuation authority. Figure 1 gives the smart fin model used in the study.

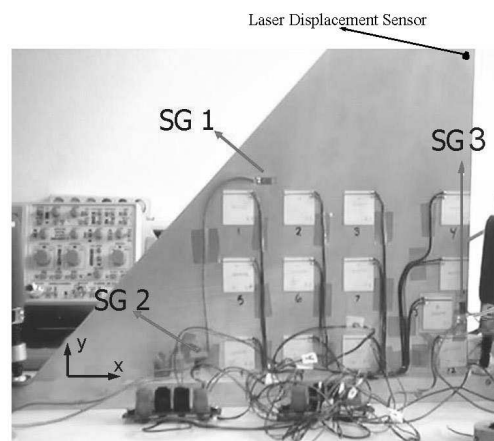


Figure1: The Smart Fin Used in the Study (SG: Strain Gage)

Table 1 gives the theoretically determined resonance frequencies together with the experimentally obtained resonance frequencies and the damping coefficients of the smart fin [10].

Table 1: Theoretically and Experimentally Obtained Resonance Frequencies and the Experimentally Found Damping Coefficients of the Smart Fin

FEM	Experimental	
$f_n(\text{Hz})$	$f_n(\text{Hz})$	Damping
14.96	14.51	4.80e-2
45.74	48.94	2.02e-1
68.25	69.43	1.79e-2

### 3. CONTROLLER DESIGN

This section gives the design of controllers via  $\mu$ -synthesis to suppress the vibration of the smart fin at its first flexural and first torsional modes (actually first two modes of the smart fin). For the controller design, first the system models were determined from relevant experimental data. The controllers were then designed, based on the experimentally identified models by defining the performance criteria and uncertainty characteristics of the identified models and the actuator limitations. The controllers were designed considering both SISO (Single-Input Single-Output) and SIMO (Single-Input Multi-Output) system models.

Unlike the previous studies conducted by Yaman et. al. [7, 8, 9, 11], in this study,  $\mu$ -synthesis method was chosen for the controller design. When compared with  $H_\infty$  control theory,  $\mu$ -synthesis method is shown to be less conservative in the controller design for the plant, which has multiple uncertainties at different locations. The reason for the conservatism is that,  $H_\infty$  synthesis method does not include the uncertainty structure in the controller design. Whereas,  $\mu$ -synthesis method allows for the introduction of the uncertainty structure in the controller design process and this lead to increase in the performance of the designed controller [12].

#### 3.1 Controller Design Based on Strain Measurements

##### 3.1.1 Controller Design for Single-Input, Single-Output System Models

In this analysis, the system models obtained from strain gage 2 and strain gage 3 (Figure 1) were considered separately for the controller design via  $\mu$ -synthesis.  $\mu$ -synthesis problem was formulated and solved by D-K iteration technique.  $\mu$ -analysis was performed for the closed loop system and the structured singular values were obtained to be less than unity. Hence, it was concluded that the designed controllers were admissible according to  $\mu$ -analysis. In addition to that, the attenuation levels at the frequency response peaks were checked by performing the open loop and closed loop frequency response simulations in Matlab (v6.5). During the design phase, it was considered that the smart fin was excited by 12 PZT's on one face and Matlab simulations were conducted accordingly. Table 2 gives the comparisons of the achieved attenuation levels, which were the ratio of a maximum open loop frequency

response to a maximum closed loop frequency response at the defined modes, for the two controllers designed for the single-input single-output system models. In the first one, the controller was designed for the system model based on the strain gage 2 measurements (i.e. controller input is the strain read from strain gage 2) and for the second one the controller was designed for the system model based on the strain gage 3 measurements (i.e. controller input is the strain read from strain gage 3).

Table 2: Comparison of the Simulated Attenuation Levels of the Smart Fin for Strain Measurement

Modes		First	Second
Controller input is Strain Gage 2	Attenuation at SG 2	3.16	5.22
	Attenuation at SG 3	3.16	4.60
Controller input is Strain Gage 3	Attenuation at SG 2	3.73	1.12
	Attenuation at SG 3	3.76	1.15

As it can be seen from Table 2 for the first mode, which is predominantly flexural both controllers performed satisfactorily. Whereas for the second mode, which is predominantly torsional, the controller designed by considering the strain gage 2 as an input had achieved better vibration suppression. These results can be explained on the grounds of the smart fin mode shapes and strain gage locations. Both strain gage 2 and strain gage 3 can primarily sense the flexural vibrations and the controllers based on them perform according to the strain signals sensed by the strain gages. Since both can sense the flexural vibrations, both can perform satisfactorily for the first mode. But because of the mode shapes of the smart fin, the strain gage 2 can also sense the vibrations of the torsional mode whereas the strain gage 3 cannot.

In the following section these two system models were combined to form a single-input multi-output system model. The aim in doing that was to achieve possible vibration suppression at all modes within the frequency range of interest for the whole structure.

### **3.1.2 Controller Design for Single-Input, Multi-Output System Model**

The single-input, single-output system models corresponding to strain gages 2 and 3 were combined and hence the control problem was re-formulated and solved. An  $\mu$ -analysis was performed for the designed controller and the  $\mu$  bounds were found to be less than unity. Also, the open loop and closed loop frequency response simulations of the smart fin were performed. The resulting simulated attenuation levels of the smart fin are given in Table 3.

Table 3: Comparison of Simulated Attenuation Levels of the Smart Fin for Strain Measurement (Controller Inputs are Strain Gages 2 & 3)

Modes	First	Second
Attenuation at SG 2	5.11	2.13
Attenuation at SG 3	5.12	1.95

The attenuation level at the first mode was improved significantly for each strain gage when both strain gages were used as controller inputs. Also in the SIMO case,

the attenuation levels became closer to each other at each strain gage location, which meant that the vibration attenuation was not achieved locally but through the whole structure.

### **3.2 Controller Design Based on Displacement Measurements**

Another application for the vibration suppression of the smart fin was conducted by considering the smart fin flexural displacement measurement as controller input. The location of the flexural displacement to be acquired by the laser beam was determined from FEM analysis [10]. The controller was designed and  $\mu$ - analysis was performed. The simulated attenuation levels were again obtained from Matlab (v6.5) frequency domain simulations and are given in Table 4.

Table 4: Simulated Attenuation Levels of the Smart Fin for Displacement Measurement

Modes	First	Second
Attenuation Levels	3.48	1.81

## **4. EXPERIMENTAL STUDIES**

The effectiveness of the developed controllers was analyzed and verified by performing the forced vibration analysis. The forced vibration experiments were conducted by using two different approaches. In the first approach, the controller implementation was performed by using a four-channel programmable controller Sensortech SS10, which was specifically designed for smart structure applications. In the second application, the designed controller was implemented by using a LabVIEW (v5.0) based program.

### **4.1. Applications Based on Strain Measurements**

Figure 2 gives the experimental set-up for strain measurement applications. The open loop and closed loop frequency response characteristics of the smart fin were analyzed.

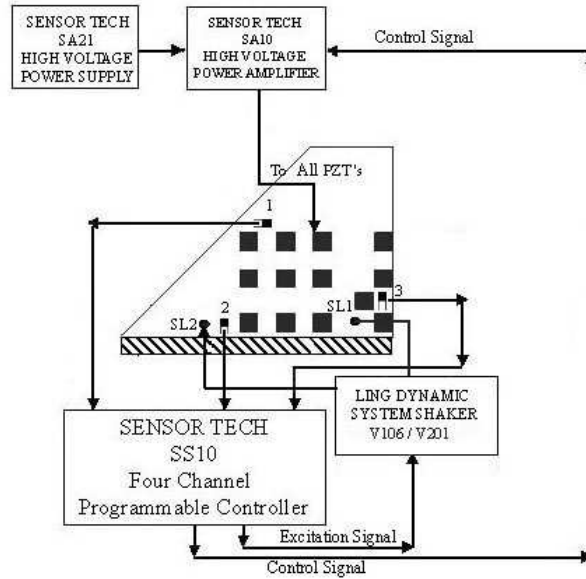


Figure 2: Experimental Setup for Strain Measurement Applications of the Smart Fin

Figures 3 to 5 give the experimental open loop and closed loop frequency responses together with Matlab closed loop simulation results for different controllers. In this analysis, the smart fin was excited by one shaker, which was located near to the strain gage 2 and was denoted as SL2 in Figure 2. In the relevant Matlab simulations only the shaker excitation was considered. Hence different simulation results were obtained as compared to the design stage analyses.

Figure 3 illustrates the frequency responses of the smart fin obtained by considering the strain gage 2 as the controller input.

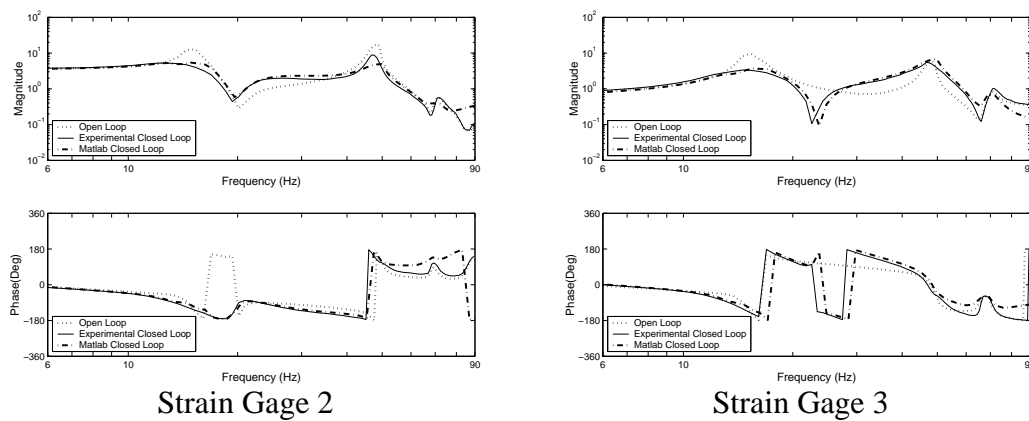


Figure 3: Open Loop and Closed Loop Forced Vibration Frequency Responses of the Smart Fin for Strain Measurement (Controller Input is Strain Gage 2)

Considerable vibration suppression was achieved at the first resonance frequency for both strain gage locations. However, at the second resonance frequency, the vibration suppression was achieved only at the strain gage 2 location. This means that the vibration could not be suppressed in the whole structure. Table 5 gives the obtained attenuation levels. As seen from the table, the attenuation levels obtained from both strain gages are close to each other at the first mode. However, at the

second mode neither the experimental nor the simulated attenuation levels at two different strain gage locations are close to each other. Hence it can be concluded that the controller designed by considering only strain gage 2 measurements as the controller input was not successful for the complete vibration suppression of the smart fin.

Table 5: Comparison of the Simulated and Experimental Attenuation Levels of the Smart Fin for Strain Measurement Undergoing a Shaker Excitation (Controller Input is Strain Gage 2)

Modes		First	Second
Simulation	Attenuation at SG 2	2.33	3.67
	Attenuation at SG 3	2.77	0.93
Experiment	Attenuation at SG 2	3.18	2.44
	Attenuation at SG 3	2.95	1.23

Figure 4 details the frequency responses of the smart fin obtained by considering the strain gage 3 as the controller input.

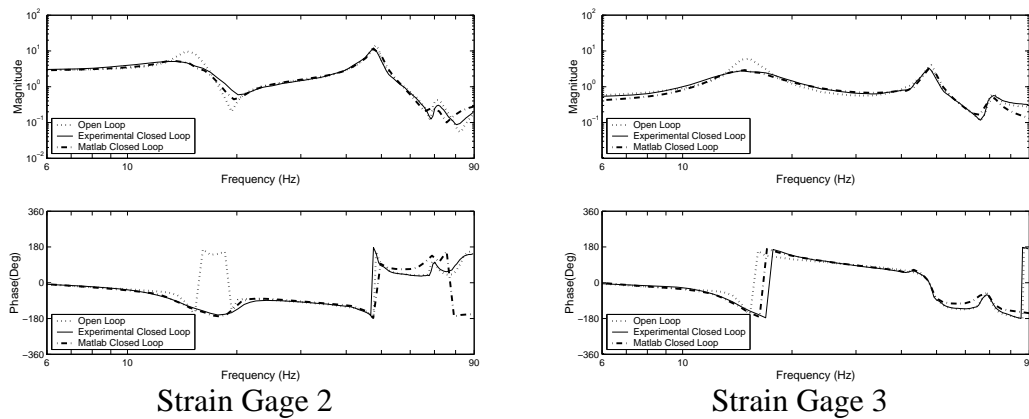


Figure 4: Open Loop and Closed Loop Forced Vibration Frequency Responses of the Smart Fin for Strain Measurement (Controller Input is Strain Gage 3)

It can be seen from Figure 4 that the vibration suppression at the first flexural and first torsional modes was achieved with the designed controller. Table 6 gives the simulated and experimentally obtained attenuation levels of the controlled system. It could be concluded that the designed controller had better vibration suppression through the whole structure, since the obtained attenuation levels at each strain gage location are close to each other. However it must be noted that, these suppression levels are small compared to the case where the controller input is strain gage 2.

Table 6: Comparison of the Simulated and Experimental Attenuation Levels of the Smart Fin for Strain Measurement Undergoing a Shaker Excitation (Controller Input is Strain Gage 3)

Modes		First	Second
Simulation	Attenuation at SG 2	2.09	1.46
	Attenuation at SG 3	2.05	1.28
Experiment	Attenuation at SG 2	2.08	1.40
	Attenuation at SG 3	2.06	1.40

Figure 5 gives the frequency responses of the smart fin obtained considering both the strain gages 2 & 3 as controller inputs.

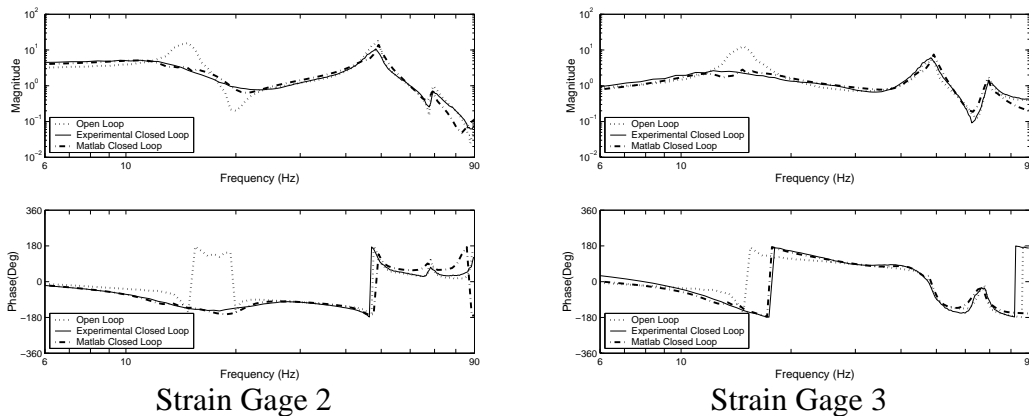


Figure 5: Open Loop and Closed Loop Forced Vibration Frequency Responses of the Smart Fin for Strain Measurement (Controller Inputs are Strain Gages 2 & 3)

The attenuation levels at each mode obtained from simulation and experiments are given in Table 7.

Table 7: Comparison of the Simulated and Experimental Attenuation Levels of the Smart Fin for Strain Measurement Undergoing a Shaker Excitation (Controller Inputs are Strain Gages 2 & 3)

	Modes	First	Second
Simulation	Attenuation at SG 2	4.90	1.27
	Attenuation at SG 3	4.47	0.80
Experiment	Attenuation at SG 2	5.86	1.97
	Attenuation at SG 3	5.66	1.13

Table 7 yields that the attenuation levels at the first mode were improved compared to the designed controllers based on SISO models.

#### 4.2 Applications Based on Displacement Measurements

Figure 6 gives the experimental set-up for displacement measurement applications. The open loop and closed loop response characteristics of the smart fin were analyzed.



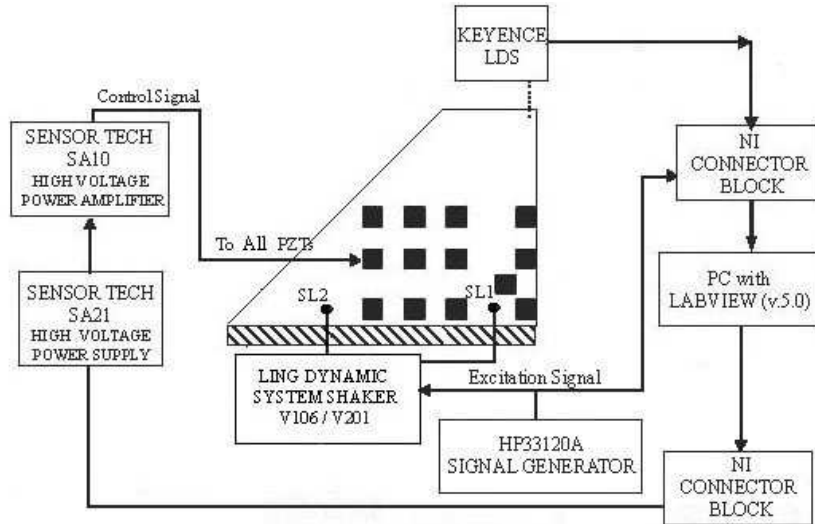


Figure 6: Experimental Setup for Displacement Measurement Applications of the Smart Fin

In Figure 7, the comparison of experimental open loop, experimental closed loop and simulated closed loop responses of the smart fin undergoing a shaker excitation is given. Table 8 gives the experimental and simulated attenuation levels of the smart fin.

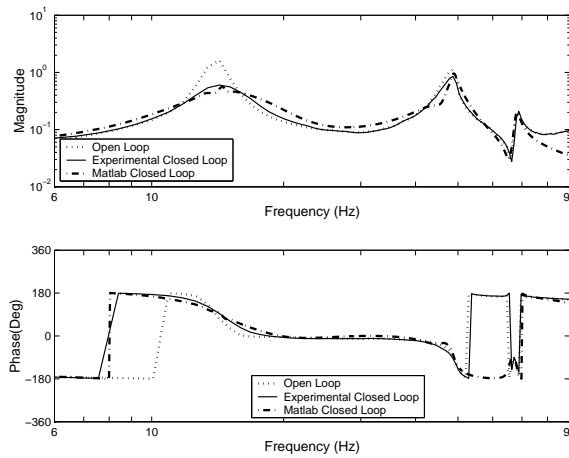


Figure 7: Open Loop and Closed Loop Forced Vibration Frequency Responses of the Smart Fin for Displacement Measurements (Controller Input is Fin Flexural Tip Displacement)

Table 8: Comparison of Simulated and Experimental Attenuation Levels of the Smart Fin for Displacement Measurement Undergoing a Shaker Excitation

Modes	First	Second
Simulation	2.90	1.16
Experiment	2.69	1.31

Considerable vibration suppression was obtained at the first two modes. Further improvement is needed for the enhancement of the attenuation level at the second

mode. This may be achieved by the modification of the structural modeling to make use of multi-input multi-output controller design.

## 5. CONCLUSIONS

This study presented the design and implementation of controllers, which were developed through  $\mu$ -synthesis for the vibration suppression of a smart fin. Two different experiments were conducted where the first one used strain gages as sensor and the second one utilized a laser displacement sensor. For both applications, the PZTs were used as actuators to suppress the vibration levels.

The controller implementations showed that the available piezoelectric actuator authority was not enough to suppress the vibrations of the smart fin. That was due to the fact that, the PZTs of only one face were effectively utilized because of the experimental limitations.

It was shown that the usage of one sensor as a controller input was not appropriate for the two dimensional structures. It is believed that a multi-input multi-output system model, which could be used by setting one of the controller output for the flexural vibration suppression and the other one for the torsional vibration suppression, may yield more satisfactory results. These studies are under way.

## 6. REFERENCES

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