OPTIMAL MAPPING OF ACTUATORS WITH CLOSED-LOOP ROBUST PERFORMANCE CONSTRAINTS

Tarkan Çalışkan[1], Afzal Suleman [2] and Yavuz Yaman [3]

1. Post-Doctoral Fellow, University of Victoria, E-mail: caliskan@uvic.ca
2. Associate Professor, University of Victoria, E-mail: suleman@uvic.ca
3. Professor, Middle East Technical University, E-mail: yyaman@metu.edu.tr

ABSTRACT

This paper presents a computationally efficient technique for the determination of the optimal size and spatial mapping of distributed actuators on a flexible structure to suppress vibrations in a H_∞ control design framework. The cost of the computations required in the H_∞ based optimization algorithm is reduced by using an efficient feasibility test. The feasibility test penalizes the candidates for the actuator size and locations resulting in the open-loop zeros remaining closer to the imaginary axes and passes the ones moving the open-loop zeros farther left of the imaginary axis. Then, by using only the candidates passing this feasibility test, optimization of the actuator size and placement can be performed using the H_∞ based design and μ analysis. The optimal mapping technique presented in this study is demonstrated on a simple finite element based model of a flexible structure consisting of a cantilevered beam with two pairs of spatially non-collocated distributed actuators and a displacement sensor.

INTRODUCTION

Sensor and actuator placement is one of the most critical aspects in controller design. Researchers have published various efforts in the development of a systematic approach for the determination of the optimal sensor and actuator placement in the control of flexible structures (Chen et al 1975, Maghami et al. 1993). Maghami et al. (1993), presented a novel approach for the optimal placement of sensors and actuators in the active vibration control of flexible space structures. In their work, the positions of the collocated sensors and actuators having negligible mass and stiffness on the flexible truss type structure are optimized to move the open-loop zeros of the system farther to the left of the imaginary axis. However, the flexible structures are generally controlled by using noncollocated sensors and actuators to avoid the performance limitations which are inevitable in collocated case. Unfortunately, this generally yields to the formation of right half plane zeros. Inniss et al (2000), showed that the existence of the right half plane zeros may result in the ill-conditioning of the zeros of the flexible structure. Furthermore, the technique is based on the open-loop characteristics of the system and neither performance nor the robustness issues are addressed.

Other researchers have attempted to determine the optimal placement of the sensors and actuators in terms of a selected closed-loop performance index, which are usually control and deformation energies within the H_∞ design framework (Arabyan et. al.1995 and 1996, Lind et al. 1997). These techniques require the search for the optimal configuration being conducted for all possible positions. Although these techniques include the closed-loop performance and robustness issues in their analysis, the large computational cost associated with the H_∞ optimal design makes this search procedure impractical for application to large flexible structures with a large number of actuators and sensors. Arabyan et al. (1999), presented a computationally efficient method to search and find the optimal configurations of the sensors and actuators. In their work, the large computational cost associated with the H_∞ optimal design is reduced by considering the optimistic lower limit, the calculation of which is less expensive than the H_∞ optimal design for all possible configurations. In this technique, H_∞ optimal design is conducted only for the candidates having optimistic lower limit less than the target deformation suppression. The optimistic lower limit is obtained by relaxing the internal stability requirement in H_∞ optimal design which may influence the robustness issues for the candidates. Furthermore, since the optimistic lower limit and standard H_∞ optimal design approaches ignore the structure of the prevalent uncertainty, the results obtained are known to yield conservative results (Zhou et al.1996, Nalbantoğlu 1996). In certain cases, this conservatism may affect the optimal mapping of the sensor and actuators.

PROPOSED METHODOLOGY

The analysis conducted for the determination of the optimal spatial configurations of the sensor and actuators for the flexible structures are either based on the open- or closed-loop behavior of the system. While the open-loop approach concentrates on the influence of the sensor actuator placement on the open-loop properties such as the transmission zeros that can directly be linked to the closed loop controller design, the closed-loop approaches deals with the placement of the controller design procedure inside the automated search algorithm. Since the open-loop approach does not require the computation of the closed loop properties, the approach reduces computational effort in the solution of the sensor actuator placement problems. However, this technique inherently excludes the performance requirements and robustness issues of the closed-loop systems in the analysis. The closed-loop approach however, includes these issues in the design process and carries out the optimal mapping for each sensor and actuator on the flexible structure. Unfortunately, this approach may result in prohibitive computational requirements.

The idea proposed here is the determination of the optimal size, spatial configurations of the distributed actuators and sensors on a flexible structure by placing the H_∞ design and μ analysis technique inside the optimization search algorithm. This process generally results in excessive computational requirements as design and analysis has to be conducted for all possible candidates in the design space. In order to reduce the computational efforts, an open-loop based feasibility test is proposed to identify the acceptable configurations on the design space. Then, the H_∞ design and μ analysis are applied to the candidates passing the feasibility test only. In this work, the robustness issues are modeled as the constraints in the optimal design.

The State-Space Representation and the feasibility test function

The aim in the system modeling is to obtain the mathematical description of the plant for the design of the control system. The system modeling technique includes the determination of the state space representation of the system. The model of the system can be found via finite element method or system identification. The dynamical model of the finite element based model for the flexible structure can be described by the second order form as,

$$[M]\{\ddot{q}\}+[D_o]\{\dot{q}\}+[K]\{q\}=[F]\{u\} \tag{1}$$

where, by defining N as the number of nodes of the finite element model and p as the number of degrees of freedoms associated with each node, [M], $[D_o]$, and [K] are $Np\times Np$ mass, damping and stiffness matrices respectively. In this representation, the vector $\{q\}_{Np\times 1}$ represents the generalized vector of displacements, $\{\dot{q}\}_{Np\times 1}$ symbolizes the generalized vector of velocities and $\{\ddot{q}\}_{Np\times 1}$ defines the generalized vector of accelerations for each node. Defining k as the number of actuators $[F]_{Np\times j}$ is the unit voltage generalized force transformation matrix from j^{th} (j=1 to k) actuator applicable to each node, and $\{u\}_{j\times 1}$ is the actuation voltage vector associated with the j^{th} actuator. Similarly, the output of the system for the i^{th} sensor (i=1 to r) can be given as,

$$\{y\}_{i\times 1}=[C_q]_{i\times Np}\{q\}_{Np\times 1}+[C_v]_{i\times Np}\{\dot{q}\}_{Np\times 1}, \quad (i=1 \text{ to } r) \tag{2}$$

here, $[C_q]$ and $[C_v]$ give the displacement and velocity output matrices respectively. The displacement and velocity output matrices represent the nodes where the response is measured. The equation of motion given in (1) should be cast into the state space form that is the one generally used in the controller design of a linear time invariant systems. The standard form of the state space representation is given as,

$$\{\dot{x}\} = [A]\{x\} + [B]\{u\}, \qquad \{y\} = [C]\{x\} \tag{3}$$

In this realization, [A] describes the system matrix, and [B] gives the input matrix, and [C] defines the output matrix. In this formulation, {u} symbolizes the vector of inputs to the system. The applications of the standard modal analysis techniques allow the determination of the state space representation in modal coordinates by considering new variable $x = \{q \quad \dot{q}\}^T$ as (Çalışkan, 2002),

$$\left\{\dot{x}\right\}_{mp\times 1} = \begin{bmatrix} [0] & [I] \\ -[\Lambda]^2 & -[D_m] \end{bmatrix}_{mp\times mp} \left\{x\right\}_{mp\times 1} + \left\{ \begin{matrix} [0] \\ [\Psi]^T_{np\times mp}[F]_{np\times j}\{u\}_{j\times 1} \end{matrix} \right\}_{mp\times 1} \quad (j = 1 \text{ to } k) \tag{4}$$

$$\{y\}_{i\times 1} = [C_q]_{i\times np}[\Psi]_{np\times mp}\{q_m\}_{mp\times 1} + [C_v]_{i\times np}[\Psi]_{np\times mp}\{\dot{q}_m\}_{mp\times 1}, \quad (i = 1 \text{ to } r) \tag{5}$$

where, Λ is an mpxmp diagonal matrix formed by the eigenvalues obtained from the solution of the generalized eigenvalue problem described by equation(1), and Ψ gives npxmp modal matrix formed by the eigenvectors obtained from the same equation. In this realization, [I] is mp×mp identity matrix, and $[D_m]$ symbolizes a mp×mp diagonal modal damping matrix. Besides, q_m symbolizes the generalized modal displacement as,

$$\{q_m\}_{mp\times 1} = [\Psi]^T_{mp\times np}\{q\}_{np\times 1} \tag{6}$$

The finite element based state-space representation given in equations (4) and (5) provides necessary and sufficient information for the feasibility test and design of the controllers that aim to suppress the vibrations due to the modes of the flexible structures.

This work considers the application of a feasibility function so as to reduce numerical computations involved in closed-loop based optimization process. Hence an appropriate open-loop based feasibility test function should link the open and closed-loop properties of the systems.

The transmission zeros of a linear time invariant systems define the asymptotic location of the closed-loop poles under high actuator gains. It has been shown that as the transmission zeros of the system moves far enough to the left-half plane the possibility of acquiring fast regulation increases (Maghami et al.,1993). Hence, a feasible candidate for the spatial variations of the actuator and sensor locations can be selected among the ones moving the transmission zeros of the system farther in the left hand plane. For non-minimum phase systems, these values can be selected by using the following open-loop objective test function.

$$J_o^k = \min_i(\mathrm{Re}(z_i)) \tag{7}$$

where z_i defines the open-loop transmission zeros of the system for the k^{th} configuration of the sensor and actuator locations. The transmission zeros of the system model given can be obtained from the solution of the generalized eigenvalue problem described by equation (8) (Maghami et al.,1993, Inniss et al. 2000).

$$\begin{bmatrix} A & -B \\ C & 0 \end{bmatrix} \left\{ \chi \right\} = s \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \left\{ \chi \right\} \tag{8}$$

<u>H_∞ Optimal Control</u>

The study utilizes H_∞ optimal control design algorithm and μ analysis techniques to achieve the closed-loop performance objectives. The effectiveness of these techniques is shown in the literature [7-10]. The standard closed loop architecture of the H_∞ controller is shown in Fig. 1. In this figure, {w}, {v}, {u}, {e}, {z} and {y} are vector valued signals. Here, {w} and {v} are the exogenous inputs, typically consisting of command signals, disturbances, and sensor noises. {u} is the control signal and {z} describes the output to be controlled and {e} symbolizes the error signals; their components typically being tracking errors, filtered actuator signal and {y} is the measured output. P(s) represents a generalized nominal transfer function of the system. In this architecture, K(s) processes the outputs and feeds back to the system. The H_∞ control problem consists of determining K(s) such that the H_∞ norm of the transfer function from {w}, {v} to {z}, {e} is minimized and the closed loop system is stable [12,13]. Unlike other conventional controllers, the uncertainties present in the system can systematically be included in the modeling. In this technique, despite of the presence of the uncertainties $\Delta(s)$ as shown in Fig. 2, the controller minimizes the ratio of the signal energies {e} to {v} (Zhou et al.1996).

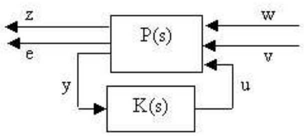

Figure 1. The closed loop architecture of the H_∞ controller

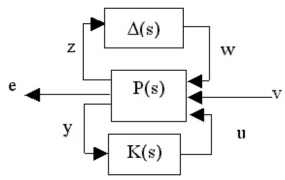

Figure 2. The modeling of the uncertainties in H_∞ controller

In H_∞ control problem, the Δ block is eliminated for the design purposes and the input-output map from $[\{w\} \ \{v\}]^T$ to $[\{z\} \ \{e\}]^T$ is expressed in lower linear fractional transformation form $F_l(P, K)$ [12] as ,

$$\begin{bmatrix} \{z\} \\ \{e\} \end{bmatrix} = F_l([P],[K]) \begin{bmatrix} \{w\} \\ \{v\} \end{bmatrix} \tag{9}$$

where, $F_l(P,K) = P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21}$ P_{ij} represents the partitioned elements of [P] (according to the dimensions of the control, measurement, disturbance and error signals) as,

$$[P] = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \tag{10}$$

The objective is to find a stabilizing controller K that minimizes the ∞-norm of $\|F_l(P,K)\|_\infty$. For an uncertainty block satisfying $\|\Delta\|_\infty < 1$, the closed loop system in Figure 2 has robust performance if $\|F_l(P,K)\|_\infty \leq 1$ is achieved [8,12]. This result, however, is conservative because it assumes that the delta block is a full block. The uncertainties in a realistic problem are due to the components of a system, and the representation of such uncertainties results in a block diagonal Δ(s). A less conservative robustness test for the closed loop system is given by examining the structured singular values (μ) of $M = F_l(P, K)$. For a given system M and an uncertainty structure, the structured singular value μ is defined (Zhou et al. 1996) as,

$$\mu_\Delta = \frac{1}{\min\{\overline{\sigma}(\Delta) : \Delta \in \Delta', \det(I - M\Delta) = 0\}} \tag{11}$$

where Δ' is the set of block diagonal matrices. In this analysis, if no $\Delta \in \Delta'$ makes (I-MΔ) singular then $\mu_\Delta(M)=0$. It has been shown that for an appropriately control design formulation the μ values less than one guaranties the robustness properties of the controller in the presence of the modeling uncertainties (Nalbantoğlu et al. 1996).

In this work, the closed loop objective for the determination of the optimal configuration of the actuators is considered as the maximization of the attenuation ratio across the frequency range of interest and the robustness issues specified by the μ values forms the constraints. Symbolizing the open and closed-loop systems by S_o and S_c respectively, the attenuation ratio corresponding to k^{th} sensor and actuator configuration is described by,

$$J_c^k = \frac{\|S_o\|_\infty}{\|S_c\|_\infty} \text{ subjected to } \mu_p \text{ and } \mu_s \leq 1 \tag{12}$$

Here μ_p and μ_s defines the performance and stability μ values respectively

Illustrative Example

The effectiveness of the technique presented is demonstrated on a finite element based simple model of a flexible structure consisting of a thin cantilevered beam with two pairs of spatially non-collocated distributed actuators and a displacement sensor. The study uses ANSYS® (v6.1) to model the flexible structure.

It has been shown that the sensor and actuator type and placement has a direct influence on the poles and zeros of the linear time invariant systems as the actuators have non negligible mass and stiffness. In these cases, the open-loop frequencies and associated mode shapes changes for each candidate that requires the modification of the system model. This effect is more prominent on the thin flexible structures consisting of piezoelectric actuators. In these structures, although the increase in the size of the actuator makes the flexible structures stiffer, it also increases the energy transmitted to the structure thereby giving a rise to the energy transmitted to the structure. That consequently increases the response of the structure. Furthermore, as the patches move away from the regions where higher strain values developed, the response decreases (Çalışkan, 2002). Fig. 3 illustrates the finite element model and the initial size and configuration of the piezoelectric patches on the flexible structure. By using the parametric modeling feature of the model that consequently allows the modeling for different size and placement of the piezoelectric actuators and the finite element based system modeling technique presented, the open-loop characteristics of the system are obtained for each candidate. During the theoretical calculations modal damping ratio is taken to be 0.03 and to secure the validity of the linear piezoelectricity and elasticity theories considered, the upper limit for the actuator size is taken to be 150mm for both of the actuators. Furthermore, massless displacement sensor placed at the mid-tip location of the flexible structure is assumed to measure vibration signals.

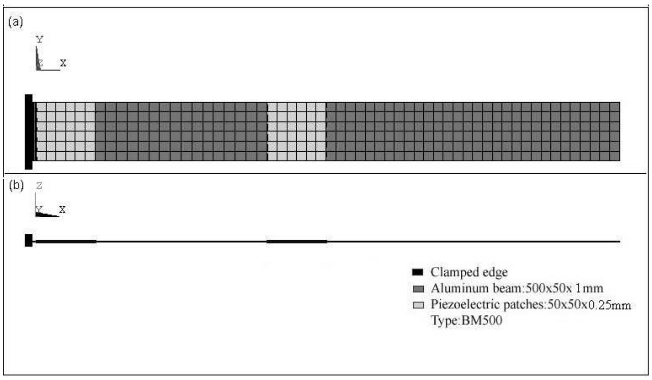

Figure 3. The finite element model and the initial size and configuration of the piezoelectric patches of the flexible structure (a. Top view, b. Side view)

In this study, 104 candidates involving different spatial configuration and size of the actuators for the determination of the optimal placement and size are considered to determine the optimal size and configuration of the piezoelectric actuators. In order to determine the number of candidate passing the test, first the test function given in equation (9) is evaluated for each candidate and the absolute value of the test function is sorted in descending order. Then, the results obtained are plotted against the index number which effectively represents the number of candidates involved. The results are shown in Fig. 4. It can be seen from the figure that the smallest number that can capture the maximum values of the test function should be in the vicinity of the first 75 candidates where the gradient of the test function is zero. The selection of smaller number influences the effectiveness of the test.

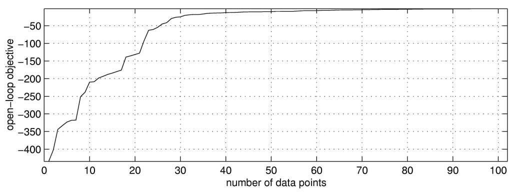

Figure 4. The feasibility test results

The H_∞ controller synthesis and μ-analysis techniques described are applied to the flexible structure with the actuator size and placement combination passing the feasibility test by using μ-Analysis and Synthesis toolbox of the commercial program Matlab®(v.6.0, 2001). During the H_∞ controller design for the flexible structure which is modeled as a single input single output system, the performance objective is selected to minimize the maximum frequency response of the first mode of the flexible structure at the sensor location. Fig. 5. shows the formulation of the closed loop control in H_∞ framework. In Fig. 5 , SYS_{flex} defines the nominal flexible structure model, δ is a complex number such that $\|\delta\| \langle 1$ and, with Δ which is the multiplication of W_{add} by δ defines the system model of the flexible structure including the uncertainties. In the modeling W_{add} defines the amplitude of the weight of additive uncertainty weight included into the system model. W_{per} represents a performance weight on the displacement sensor to achieve the performance objective. These weights are adjusted to achieve attenuation in the peak frequency response of the closed loop systems involving different actuator configurations.

In this study, to limit the actuator command signal in the control design process to 250 volts W_{act} in Fig. 5 is chosen as 1/250. The weights on the disturbance input, W_{dist} is taken to be 1 indicating that the input disturbance is expected to be on the same order of magnitude as the controller signals. The displacement signal is considered to have a signal to noise ratio of 100. Therefore, W_{noise} in Fig. 5. is taken as a diagonal matrix with 0.01 as the diagonal elements. The absorption of the weights into the system model results in the standard closed loop formulation of the H_∞ controller shown in Fig. 2

The purpose in the controller design is to minimize displacement signal in the low frequency range, while not exciting the unmodeled high frequency modes (Zhou et al.1996, Nalbantoğlu 1996). Fig. 6 shows the magnitude plot of the weightings together with the open-loop transfer function of the initial configuration. It can be seen from the figure that, as the frequency increases W_{add} increases indicating a better system model at low frequencies. The performance weight is selected by considering the results of the feasibility test. The comparison of the W_{per} and the frequency response of the flexible structure with the initial actuator position and size The initial configuration (x_1=3, x_2=200,s_1=50, s_2=50, in mm) are also shown in Fig. 6. The application of this weight yields the minimization of the displacement at low frequencies while making minimal changes at high frequencies.

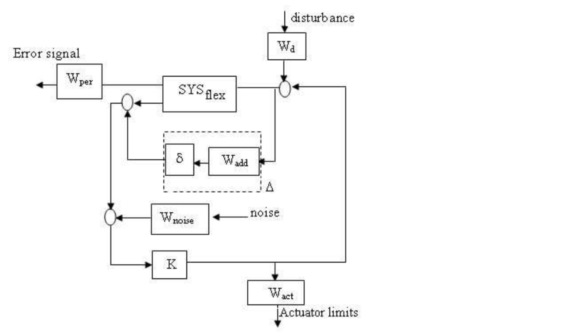

Figure 5. The control problem formulation

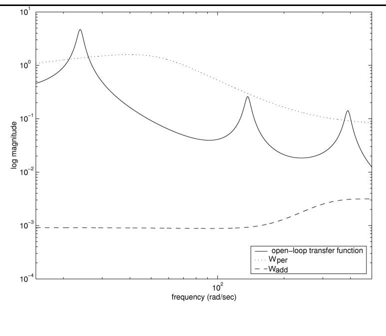

Figure 6. The comparison of the open-loop Transfer function, W_{add} and W_{per}

The utilization of these weights together with the system models obtained for the candidates passing the feasibility test results in the standard H_∞ control design formulation shown in Fig. 1. In this work, the robustness issues are addressed through the application of μ analysis for each feasible candidate. In order to confirm the validity of the proposed technique, H_∞ control design and μ analysis conducted for all candidates and the results are plotted against the candidate number that includes information for each design variable in Fig. 7. It appears from Fig. 8 that the feasibility test effectively captures the acceptable candidates which have the largest attenuation ratios by using 75 candidates. This allows the application of formal optimization techniques to 75 acceptable configurations instead of 104. The μ analysis is also conducted for the candidates passing the test and the results are plotted in Fig. 9. The existence of the μ values less than one indicates that the optimal controllers have robust performance in the presence of the modeling uncertainties.

Indicating the positions of the first actuators by x_1 and the second by x_2 and the length of the first and the second actuators by s_1, s_2 respectively, Fig. 10 gives the comparison of the open and closed-loop frequency response functions corresponding to different actuator size and placement values. These candidates are the ones corresponding to the initial configuration and maximum attenuation ratios. It can be seen from the figure that significant improvements in the attenuation levels can be obtained by considering different actuator size and configurations for the model considered. The attenuation ratios achieved at the first mode of vibration for the initial configuration and the maximum closed loop objective functions values are 3.75, and 13.05 respectively.

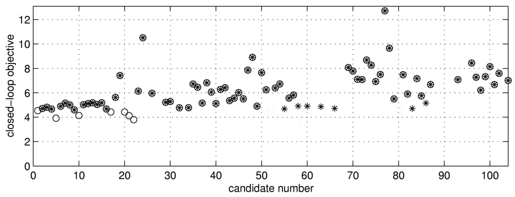

Figure 8. The comparison of the open-loop and closed-loop objective functions (o: candidates passing the feasibility test,*: closed-loop objective)

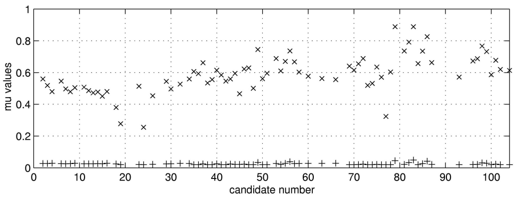

Figure 9. The μ analysis results for the candidates passing the feasibility test (x: μ_p, +: μ_s)

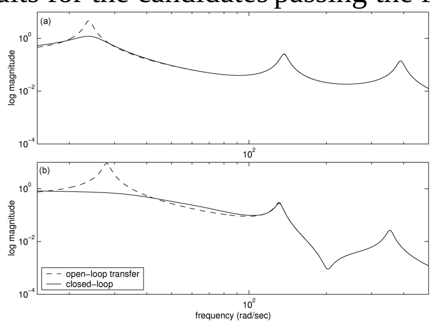

Figure 10. The comparison of the open and closed-loop responses (a: initial configuration, b: maximum attenuation ratio; x_1=9, x_2=250 s_1=150, s_2=50, in mm)

CONCLUSIONS AND FUTURE WORK

A computationally feasible approach for the actuator size and placement in the control of flexible structures has been demonstrated. The computations required for the H_∞ control design algorithm and μ analysis techniques have been reduced by computationally inexpensive open-loop based feasibility test. Since the approach effectively determines the acceptable configurations in the design space and allows appropriate specifications of the weightings in H_∞ control design and μ analysis techniques, the approach is feasible mainly for the problems involving high order models having large number of actuators and sensors. The technique presented is expected to improve the number of computations involved in the optimal mapping with exhaustive search or genetic algorithms. Although the technique demonstrated on the actuator size and placement problem on the simple beam, the technique may be extended to more complex actuator-sensor structural systems.

ACKNOWLEGMENTS

Author[1] is supported by The Scientific and Technical Research Council of Turkey within the scope of NATO science fellowship programme, and also the NSERC Strategic Grant no. STPGP 246515-01. The support given is gratefully acknowledged.

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