

Spatial Control of a Smart Beam

Omer Faruk KIRCALI^{+,†}, Yavuz YAMAN⁺, Volkan NALBANTOGLU⁺,
Melin SAHIN⁺, Fatih Mutlu KARADAL⁺, Fatma Demet ULKER^{*}

⁺Department of Aerospace Engineering, Middle East Technical University, Ankara, TURKEY

[†]Defence Technologies Inc., Ankara, TURKEY

^{*}Mechanical and Aerospace Engineering Department, Carleton University, CANADA

Abstract: This study presents the design and implementation of a spatial H_∞ controller for the active vibration control of a smart beam. The smart beam was modeled by assumed-modes method that results in a model including large number of resonant modes. The order of the model was reduced by direct model truncation and the model correction technique was applied to compensate the effect of the contribution of the out of range modes to the dynamics of the system. Additionally, spatial identification of the beam was performed, by comparing the analytical and experimental system models, in order to determine the modal damping ratios of the smart beam. Then, the spatial H_∞ controller was designed and implemented to suppress the first two flexural vibrations of the smart beam.

Keywords: smart beam, assumed-modes, spatial system identification, spatial H_∞ control

1. Introduction

The vibration is an important phenomenon for the lightweight flexible aerospace structures. That kind of structures may be damaged under any undesired vibrational load. Hence, minimizing the structural vibration is necessary and this is achieved by means of a control mechanism. The usage of smart materials, as actuators and/or sensors, has given the opportunity to be used as a control mechanism.

The smart structure is a structure that can sense external disturbance and respond to that with active control in real time to maintain mission requirements ^[1]. Active vibration control of a smart structure requires an accurate system model of the structure. Modeling smart structures may require the modeling of both passive structure and the active parts. The governing differential equations of motion of the smart structures can be solved by analytical methods, such as assumed-modes method or finite element method ^[2]. Crawley and de Luis ^[3] presented an analytical modeling technique to show that piezoelectric actuators can be used to suppress some modes of vibration of a cantilevered beam. Caliskan ^[1] presented modeling of the smart structures by finite element modeling technique. Nalbantoglu ^[4] showed that experimental system identification techniques can also be applied on flexible structures to

identify the system more accurately. The system model of a flexible structure has large number of resonant modes; however, general interest in control design is only on the first few ones. Hence, reducing the order of the system model is often required^[5,6]. A common approach is the direct model reduction. However, removing the higher modes directly from the system model perturbs the zeros of the system^[7]. Therefore, in order to minimize the model reduction error, a correction term, including some of the removed modes, should be added to the truncated model^[7,8].

Today, robust stabilizing controllers designed in respect of H_∞ control technique are widely used on active vibration control of smart structures. Yaman et al.^[9,10] showed the effect of H_∞ controller on suppressing the vibrations of a smart beam due its first two flexural modes. Similar work is done for active vibration control of a smart plate, and usage of piezoelectric actuators on vibration suppression with H_∞ controller is successfully presented^[11]. Ulker^[12] showed that, beside the H_∞ control technique, μ -synthesis based controllers can also be successfully used to suppress the vibrations of smart structures.

Whichever controller design technique is applied, the suppression is preferred to be achieved over the entire structure rather than at specific points, since the flexible structures are usually distributed parameter systems. Moheimani and Fu^[13] and Moheimani et al.^[14] introduced spatial H_2 norm and H_∞ norm concepts in order to meet the need of spatial vibration control, and simulation based results of spatial vibration control of a cantilevered beam were presented. Moheimani et al.^[15] studied spatial feedforward and feedback controller design, and presented illustrative results. They also showed that spatial H_∞ controllers could be obtained from standard H_∞ controller design techniques. Halim^[16,17] studied the implementation of spatial H_2 and H_∞ controllers on active vibration control and presented quite successful results. However these works were limited to a beam with simply-supported boundary condition.

This paper aims to present design and implementation of a spatial H_∞ controller on active vibration control of a cantilevered smart beam.

2. Modeling of the smart beam

Consider the cantilevered smart beam given in Fig. 1. The structural properties are given at Table 1. The smart beam consists of a passive aluminum beam (507mmx51mmx2mm) with symmetrically surface bonded eight SensorTech BM500 type PZT (Lead-Zirconate-Titanate) patches (25mmx20mmx0.5mm), which are used as actuators. The beginning and end locations of the PZT patches along the length of the beam are denoted as r_1 and r_2 , where the patches are accepted as optimally placed^[1]. The subscripts b and p indicate the beam and

PZT patches, respectively. Note that, despite the actual length of the beam is 507mm, the effective length reduces to 494mm since it is clamped in the fixture.

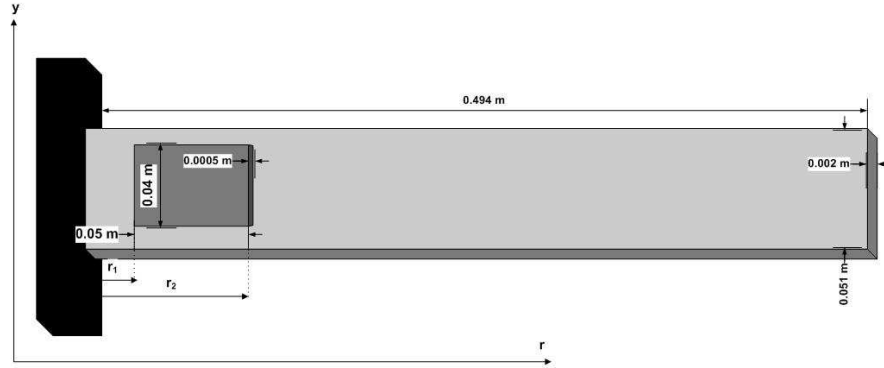


Fig.1: The smart beam model used in the study

Table 1: Properties of the Smart Beam

	Beam	PZT
Length, m	$L_b = 0.494$	$L_p = 0.05$
Width, m	$w_b = 0.051$	$w_p = 0.04$
Thickness, m	$t_b = 0.002$	$t_p = 0.0005$
Density, kg/m^3	$\rho_b = 2710$	$\rho_p = 7650$
Young's Modulus, GPa	$E_b = 69$	$E_p = 64.52$
Cross-sectional Area, m^2	$A_b = 1.02 \cdot 10^{-4}$	$A_p = 0.2 \cdot 10^{-4}$
Second Moment of Area, m^4	$I_b = 3.4 \cdot 10^{-11}$	$I_p = 6.33 \cdot 10^{-11}$
Piezoelectric charge constant, m/V	-	$d_{31} = -175 \cdot 10^{-12}$

Analytical modeling of the smart beam is performed by assumed-modes method, which represents the deflection of the beam by means of a series solution:

$$y(r, t) = \sum_{i=1}^N \phi_i(r) q_i(t) \quad (1)$$

where $\phi_i(r)$ are admissible functions which satisfy the geometric boundary conditions of the passive beam, $q_i(t)$ are time-dependent generalized coordinates, r is the longitudinal axis and t is time. Assumed-modes method uses this solution to obtain approximate system model of the structure with the help of energy expressions [2]. The admissible functions are selected as the eigenfunctions of the passive cantilevered beam with the same structural properties as:

$$\phi_i(r) = L_b \{ \cosh \beta_i r - \cos \beta_i r - \alpha_i (\sinh \beta_i r - \sin \beta_i r) \} \quad (2)$$

where

$$\alpha_i = \frac{\cos \beta_i L_b + \cosh \beta_i L_b}{\sin \beta_i L_b + \sinh \beta_i L_b} \quad (3)$$

where eigenfunctions satisfy the orthogonality conditions given below:

$$\int_0^{L_b} \rho_b A_b \phi_i(r) \phi_j(r) dr = \rho_b A_b L_b^3 \delta_{ij} \quad (4)$$

$$\int_0^{L_b} E_b I_b \frac{d^2 \phi_i(r)}{dr^2} \frac{d^2 \phi_j(r)}{dr^2} dr = \rho_b A_b L_b^3 \omega_i^2 \delta_{ij} \quad (5)$$

After some mathematical manipulations including the effect of PZT patches on passive beam^[18], one can reach the input and output relation of the system dynamics as a transfer function from applied voltage to the deflection of the smart beam in the frequency domain as:

$$G_N(s, r) = \sum_{i=1}^N \frac{P_i \phi_i(r)}{s^2 + 2\xi_i \omega_i s + \omega_i^2} \quad (6)$$

Equation (6) implies the analytical system model of the smart beam shown in Fig.1 including N number of resonant modes, where P_i is:

$$P_i = \frac{C_p [\phi_i'(r_2) - \phi_i'(r_1)]}{\{\rho A L^3\}_c} \quad (7)$$

where $\{\rho A L^3\}_c = \rho_b A_b L_b^3 + 2\rho_p A_p L_p^3$. The geometric constant $C_p = E_p d_{31} w_p (t_p + t_b)$ is related with the bending moment of PZT patches exerted on the beam^[19].

3. Model Correction

The analytical model of the smart beam consists of large number of resonant modes. However, in control design we deal with only the first few vibration modes of the smart beam. Hence, the full order model should be truncated to a lower order one. So the truncated model including first M number of modes can be expressed as:

$$G_M(s, r) = \sum_{i=1}^M \frac{P_i \phi_i(r)}{s^2 + 2\xi_i \omega_i s + \omega_i^2} \quad (8)$$

where $M \ll N$. This truncation may cause error due to the removed higher order modes, which can be expressed as:

$$\begin{aligned} E(s, r) &= G_N(s, r) - G_M(s, r) \\ &= \sum_{i=M+1}^N \frac{P_i \phi_i(r)}{s^2 + 2\xi_i \omega_i s + \omega_i^2} \end{aligned} \quad (9)$$

In order to reduce the truncation error, a correction term should be added to the truncated model [8].

$$G_c(s, r) = G_M(s, r) + \sum_{i=M+1}^N \phi_i(r)k_i \quad (10)$$

where the optimal and general expression of constant k_i is [20]:

$$k_i^{opt} = \frac{1}{4\omega_c\omega_i} \frac{1}{\sqrt{1-\xi_i^2}} \ln \left\{ \frac{\omega_c^2 + 2\omega_c\omega_i\sqrt{1-\xi_i^2} + \omega_i^2}{\omega_c^2 - 2\omega_c\omega_i\sqrt{1-\xi_i^2} + \omega_i^2} \right\} P_i \quad (11)$$

Consequently, the illustrative system models of the smart beam are shown in Fig.2 for different points of interest along the beam. The full order model includes first 50 modes of the beam, i.e. $N=50$, whereas the truncated one includes only 2, i.e. $M=2$. The system is considered as undamped.

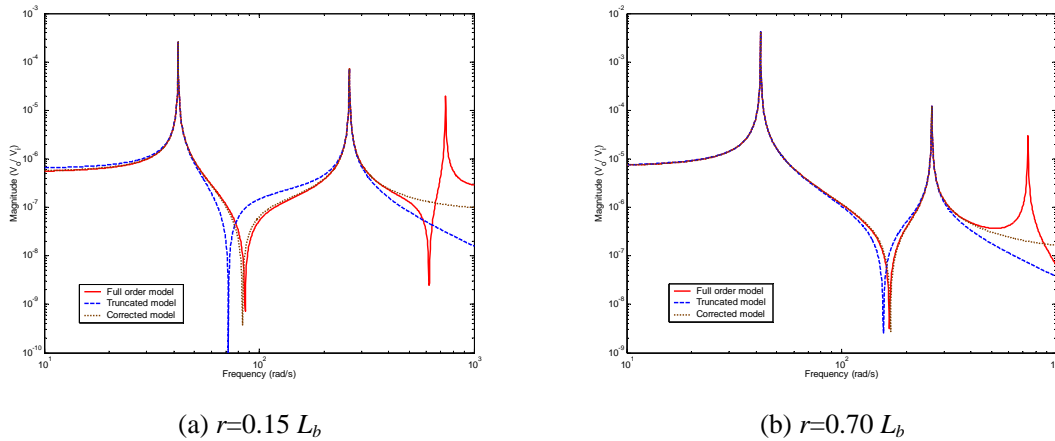


Fig.2: Frequency response of the smart beam at different points

The magnitude of the frequency function shows the input output relation of the system, therefore it is unitless and presented as V_o/V_i , assuming V_o is the output voltage defined as the deflection of the beam measured by laser sensor and converted to a voltage value, and V_i is the input voltage to the system.

4. Spatial System Identification

Experimental system identification, in collaboration with the analytical model, helps one to expose more accurate spatial characteristics of the structure. The frequency analysis lets one to obtain the transfer function of the system [4] and clarify the resonance frequency values.

Additionally, comparing the experimental model with the analytical one leads to determine the modal damping terms and the uncertainty on resonance frequencies ^[21].

Consider the smart beam of interest shown in Fig.3, where the PZTs are used as the actuators and a Keyence LB-1201(W) LB-300 laser displacement sensor is used as the sensor.

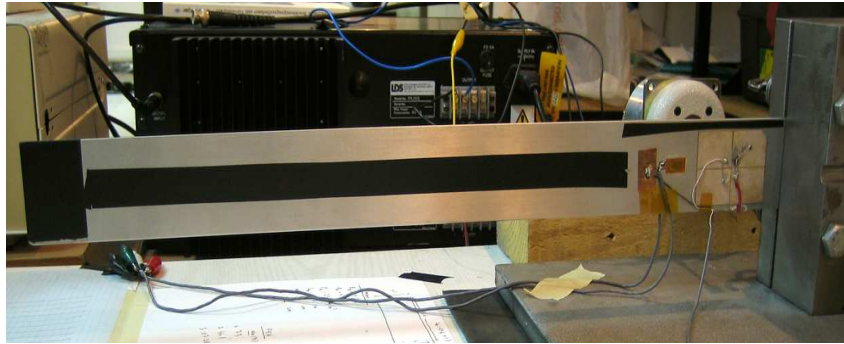


Fig.3: The smart beam used in the study

The smart beam was excited with sinusoidal chirp signal of amplitude 5V within bandwidth 0.1-60 Hz, which covers first two flexural modes of the smart beam, and the response of the smart beam was acquired via laser displacement sensor from 17 different measurement points which are separated by 0.03 m interval from tip to root of the beam, i.e. $0.99L_b$, $0.93L_b$, ...etc. Assuming that the patches are relatively thin compared to the passive aluminum beam, our model can be considered as 1-D single input multi output system, where all the vibration modes are flexural modes. The experimental setup is shown in Fig.4.

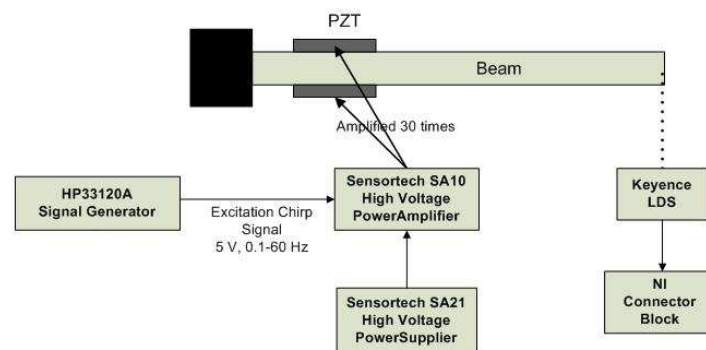


Fig.4: Experimental setup for the spatial system identification of the smart beam

The applied voltage and the time response of the smart beam is shown in Fig.5. The experimental and analytical models for 17 different measurement locations over the beam were compared and modal damping ratios were tuned till the magnitude of the analytical and

experimental frequency responses at resonance frequencies match. As an example, the frequency responses for point $r=0.99L_b$ is shown in Fig.6.

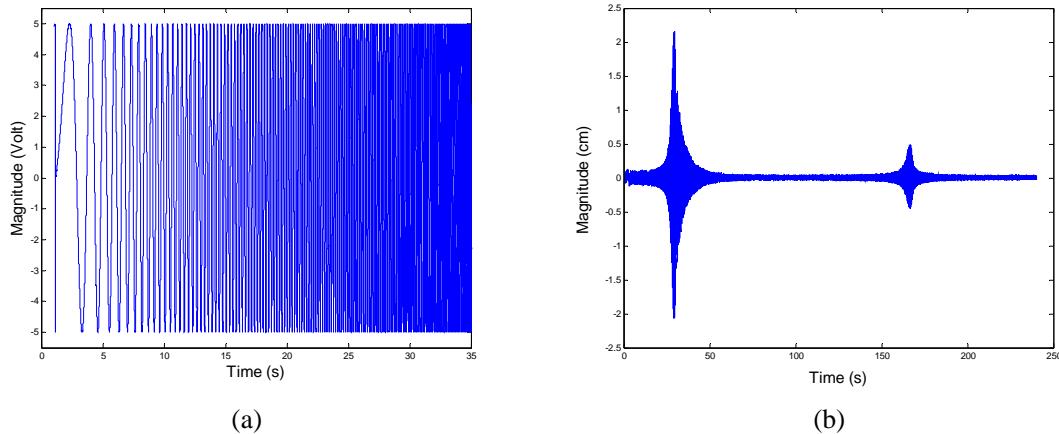


Fig.5: (a) Applied voltage (b) Time response of the smart beam measured at $r=0.99L_b$

The uncertainty on resonance frequencies and modal damping ratios can also be determined by spatial system identification. There are different methods that can be applied to determine the uncertainty and improve the values of the parameters ω and ζ such as boot-strapping ^[22]. However, in this study we define the uncertainty as the standard deviation of the parameters. The final values of first two resonance frequencies and modal damping ratios of the smart beam are presented at Table 2.

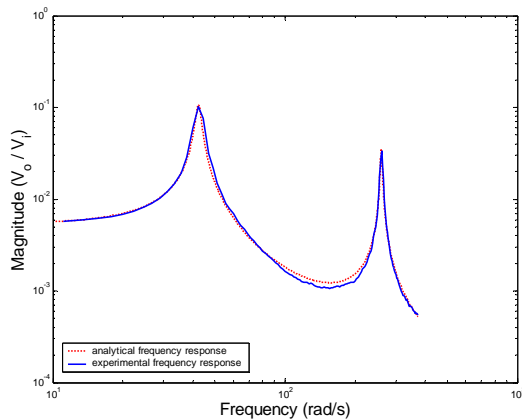


Fig.6: Experimental and analytical frequency responses of the beam

Table 2: Mean and standard deviation of the first two resonance frequencies and modal damping ratios

	$\omega_1(Hz)$	$\omega_2(Hz)$	ζ_1	ζ_2
Mean	6.742	41.308	0.027	0.007
Standard Deviation	0.009	0.166	0.002	0.001

5. Spatial H_∞ control of the smart beam

The spatial H_∞ control problem is to design a controller so as the ratio of the spatial energy of the system output to that of the disturbance signal is minimized ^[23]. Spatial H_∞ controllers could be obtained from standard H_∞ controller design technique ^[15]. Due to number of pages limitation, we will only present the results in this section and for more detailed explanation the reader is referred to references ^[15] and ^[23].

Assume that, the disturbance is entering the system through the same channel as the controller output, the state-space representation of the system is:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1 w(t) + B_2 u(t) \\ y(t, r) &= \begin{bmatrix} \Pi \\ 0 \end{bmatrix} x(t) + \begin{bmatrix} \Theta_1 \\ 0 \end{bmatrix} w(t) + \begin{bmatrix} \Theta_2 \\ \kappa \end{bmatrix} u(t) \\ \tilde{y}(t, r_L) &= C_2 x(t) + D_3 w(t) + D_4 u(t) \end{aligned} \quad (12)$$

where x is the state vector, w is the disturbance input, u is the control input, $y(r, t)$ is the performance output, $\tilde{y}(t, r_L)$ is the measured output at location $r_L=0.99L_b$, A is the state matrix, B_1 and B_2 are the input matrices from disturbance and control actuators respectively, Π is the output matrix of error signals, C_2 is the output matrix of sensor signals, Θ_1 , Θ_2 , D_3 and D_4 are the correction terms from disturbance actuator to error signal, control actuator to error signal, disturbance actuator to feedback sensor and control actuator to feedback sensor respectively and κ is the control weight designating the level of vibration suppression. Control weight prevents the controller having excessive gain and smaller κ results in higher level of vibration suppression. However, optimal value of κ should be determined in order not to neutrally stabilize the system. The state space representation variables are as follows:

$$\begin{aligned} A &= \begin{bmatrix} 0_{2 \times 2} & I_{2 \times 2} \\ -diag(\omega_1^2, \omega_2^2) & -diag(2\xi_1 \omega_1, 2\xi_2 \omega_2) \end{bmatrix}_{4 \times 4}, B_1 = B_2 = [0 \ 0 \ P_1 \ P_2]^T, \\ C_2 &= [\phi_1(r_L) \ \phi_1(r_L) \ 0 \ 0]^T, D_3 = D_4 = \sum_{i=M+1}^N \phi_i(r_L) k_{ri}^{opt} \\ \Pi &= \begin{bmatrix} L_b^3 & 0 & 0 & 0 \\ 0 & L_b^3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}_{5 \times 4}, \Theta_1 = \Theta_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & \sqrt{\sum_{i=M+1}^N (k_{ri}^{opt})^2} \end{bmatrix}_{1 \times 5}^T \end{aligned}$$

Control weight κ is selected as 0.00235. Note that the size of the state space variables strictly depend on the order of the truncated model, e.g. in our case $M=2$ so the size of state matrix A is 4×4 . The experimental setup for implementation of spatial controller is shown in Fig.7. Firstly, the free vibration suppression of the smart beam is studied. The smart beam is given an initial 5 cm tip deflection and the suppression of the free vibration of the smart beam is presented in Fig.8. Then, the forced vibration suppression of the smart beam is conducted. The smart beam is vibrated at its first two resonance frequencies by the help of a shaker and the effect of spatial controller on suppression of the forced vibrations are presented in Fig.9 (a) and (b).

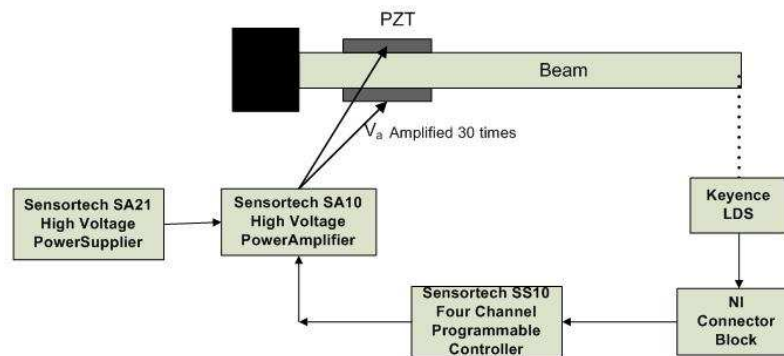


Fig.7: Experimental setup for control of the smart beam

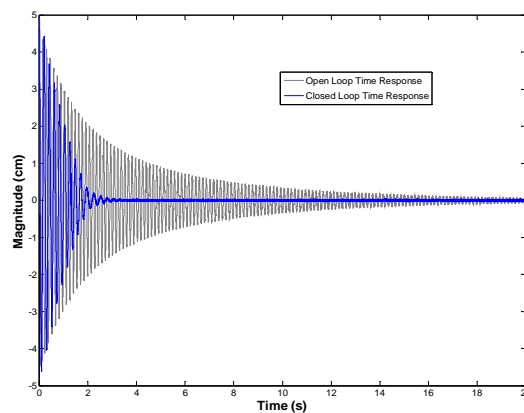


Fig.8: Free vibration of the smart beam

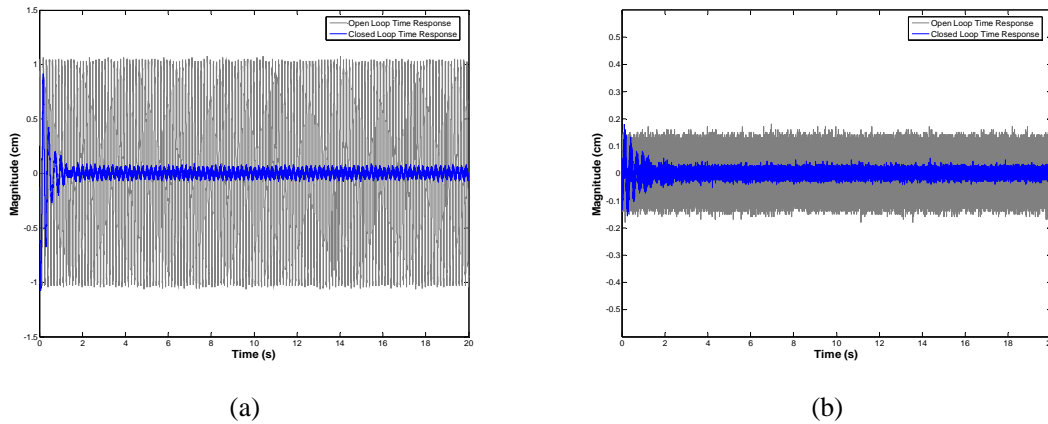


Fig.9: (a) Forced vibration of the smart beam at its first resonance (b) Forced vibration of the smart beam at its second resonance

6. Conclusion

In this paper, the design and implementation of a spatial H_∞ controller was presented for suppressing the first two flexural vibrations of a smart beam. The system model of the smart beam was obtained analytically and then improved experimentally. The free and forced vibrations of the smart beam were suppressed successfully. Further studies including comparison of pointwise and spatial controllers are in progress.

References

- [1] Caliskan T., Smart Materials and Their Applications in Aerospace Structures, *PhD Thesis, Middle East Technical University*, September 2002.
- [2] Meirovitch L., Elements of Vibration Analysis, the McGraw-Hill Company, 1986.
- [3] Crawley E.F., Louis J., Use of Piezoelectric Actuators as Elements of Intelligent Structures, *AIAA Journal*, Vol. 125, No. 10, pp. 1373-1385, October 1989.
- [4] Nalbantoglu V., Robust Control and System Identification for Flexible Structures, *Ph. D. Thesis, University of Minnesota*, 1998.
- [5] Hughes P.C., Skelton R.E., Modal Truncation for Flexible Spacecraft, 1981, *Journal of Guidance and Control*, Vol. 4 No.3.
- [6] Moheimani S.O.R., Pota H.R., Petersen I.R., Spatial Balanced Model Reduction for Flexible Structures, *Proceedings of the American Control Conference*, pp. 3098-3102, Albuquerque, New Mexico, June 1997.

- [7] Clark R.L., Accounting for Out-Of-Bandwidth Modes in The Assumed Modes Approach: Implications on Colocated Output Feedback Control, Transactions of the ASME, *Journal of Dynamic Systems, Measurement, and Control*, Vol. 119, pp. 390-395, September 1997.
- [8] Moheimani S.O.R., Minimizing The Effect of Out of Bandwidth Modes in Truncated Structure Models, Transactions of the ASME, *Journal of Dynamic Systems, Measurement, and Control*, Vol. 122, pp.237-239, March 2000.
- [9] Yaman Y., Caliskan T., Nalbantoglu V., Prasad E., Waechter D., Yan B., Active Vibration Control of a Smart Beam, *Canada-US CanSmart Workshop on Smart Materials and Structures*, 2001 Montreal, Canada Proceedings pp:137-147.
- [10] Yaman Y., Ulker F. D., Nalbantoglu V., Caliskan T., Prasad E., Waechter D., Yan B., Application of H_∞ Active Vibration Control Strategy in Smart Structures, *AED2003, 3rd International Conference on Advanced Engineering Design*, Paper A5.3, Prague, Czech Republic, 01-04 June, 2003.
- [11] Yaman Y., Caliskan T., Nalbantoglu V., Ulker F. D., Prasad E., Waechter D., Yan B., Active Vibration Control of Smart Plates by Using Piezoelectric Actuators, *ESDA2002, 6th Biennial Conference on Engineering Systems Design and Analysis*, Paper APM-018, Istanbul, Turkey, July 8-11, 2002.
- [12] Ulker F.D., Active Vibration Control of Smart Structures, *M.S. Thesis, Middle East Technical University*, September 2003.
- [13] Moheimani S.O.R., Fu M., Spatial H_2 Norm of Flexible Structures and its Application in Model Order selection, *International Proceedings of 37th IEEE Conference on Decision and Control*, Tampa Florida, USA, 1998.
- [14] Moheimani S.O.R., Pota H.R., Petersen I.R., Spatial Balanced Model Reduction for Flexible Structures, *Proceedings of the American Control Conference*, pp. 3098-3102, Albuquerque, New Mexico, June 1997.
- [15] Moheimani S.O.R., Petersen I.R., Pota H.R., Broadband Disturbance Attenuation over an Entire Beam, 1999, *Journal of Sound and Vibration*, 227(4): 807-832.
- [16] Halim D., Moheimani S.O.R., Spatial H_2 Control of a Piezoelectric Laminate Beam: Experimental Implementation, July 2002, *IEEE Transactions on Control System Technology*, Vol. 10, No: 4.
- [17] Halim D., Moheimani S.O.R., Experimental Implementation of Spatial H_∞ Control on a Piezoelectric Laminate Beam, September 2002, *IEEE/ASME Transactions on Mechatronics*, Vol. 7, No: 3.

-
- [18] Kircalı O.F., Yaman Y., Nalbantoglu V., Sahin M., Karadal F.M., Spatial System Identification of a Smart Beam by Assumed-Modes Method and Model Correction, *Kayseri VI. Aeronautics Symposium*, May 12-14, Nevsehir, Turkey (in Turkish).
- [19] Pota H.R., Alberts T.E., Multivariable Transfer Functions for a Slewing Piezoelectric Laminate Beam, *IEEE Systems Engineering*, 1992.
- [20] Halim D., Vibration Analysis and Control of Smart Structures, August 2002, *PhD. Thesis, School of Electrical Engineering and Computer Science, University of Newcastle*, Australia.
- [21] Reinelt W., Moheimani S.O.R., Identification of a Flexible Beam, *In Proceedings of the 8th International Mechatronics Conference*, Enschede, Netherlands, 2002.
- [22] Kircalı O.F., Yaman Y., Nalbantoglu V., Sahin M., Karadal F.M., Ulker F.D., Spatial System Identification of a Smart Beam, *Kayseri VI. Aeronautics Symposium*, May 12-14, Nevsehir, Turkey (in Turkish).
- [23] Moheimani S.O.R., Halim D., Fleming A.J., Spatial Control of Vibration. Theory and Experiments, 2003, World Scientific Publishing Co. Pte. Ltd.