Active Vibration Suppression of a Smart Beam via $PI^{\lambda}D^{\mu}$ Control

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Abstract: In this study, a $PI^{\lambda}D^{\mu}$ controller was designed for the active vibration suppression of a smart beam equipped with PZT (Lead-Zirconate-Titanate) patches. The smart beam is a cantilever aluminium beam having eight symmetrically located surface-bonded piezoelectric patches. In this particular application, a group of PZT patches closed to the root of the beam was used as actuators and from the remaining ones a patch was nominated as a sensor. All the actuators were used in bimorph configuration. $PI^{\lambda}D^{\mu}$ controllers were known to provide better flexibility in adjusting the gain and phase characteristics than conventional integer-order PID controllers. The parameters of $PI^{\lambda}D^{\mu}$ controllers are composed of the proportionality constant, integral constant, derivative constant, derivative order and integral order; hence its design is inevitably more complicated than that of a PID controller. First, the optimization problem of the regulator controller's required parameters was established by using the transfer function of the smart beam. That transfer function was experimentally obtained by means of using aforementioned group of sensor/actuator piezoelectric patches. The $PI^{\lambda}D^{\mu}$ controller was then considered by using a fourth degree approach of continued fraction expansion (CFE) method. Following this, the optimization problem of finding the $PI^{\lambda}D^{\mu}$ controller's parameters was solved by using both the MATLAB Optimization Toolbox and an optimization model, which has been developed in house by using MATLAB Simulink package program. The simulation results obtained in time domain demonstrated that the designed controller successfully suppressed the vibration levels of the smart beam for the first two flexural modes.

Keywords: Smart beam, PZT patches, fractional order control, vibration suppression

1. Introduction

Lightweight flexible aerospace structures require a proper active control mechanism to attenuate the vibration levels they experience, in order to preserve their structural integrity. Recent technological developments on piezoelectric materials have successfully paved ways for active vibration control applications. The piezoelectric materials offer cost effective solutions when used in the active vibration suppression of flexible structures due to their direct (i.e. sensing) and converse (i.e. actuation) piezoelectric effects.

During the previous studies conducted in the Department of Aerospace Engineering of Middle East Technical University[1], the structural modelling characteristics and active suppression of in-vacuo vibrations of the finite and flat aluminium cantilever beam-like (called as smart beam) structures were analyzed. Caliskan [2] presented modelling of the smart structures by using finite element modelling technique. Nalbantoglu [3] showed that the experimental system identification techniques could also be applied on these flexible structures to identify the system more accurately. The system model of a flexible structure has large number of resonant modes; however, general interest in control design is only on the first few ones by reducing the order of the system model [4]. The effect of H_∞ controller on suppressing the vibrations of a smart beam due its first two flexural modes were presented by Yaman et al. [5,6]. Further studies continued by Kircali et al. [7.8] on active vibration control of a smart beam by using a spatial approach. And recently, active vibration suppression of a smart beam via self-sensing piezoelectric actuator was studied by Aridogan et al.[9]. In all these studies, the smart beam was considered as an integer order plant for the active vibration control applications. The usage of non-integer order controllers for integer order plants provides better flexibility in adjusting the gain and phase characteristics than that of in integer order controllers [10,11,12]. This flexibility makes fractional order control a powerful tool in designing of robust control system with less controller parameters to tune. Additionally, the introduction of the fractional order control makes a more straightforward trade-off between the stability and other control specifications to achieve a better performance. Therefore, $PI^{\lambda}D^{\mu}$ controller, which is an important member of PID controllers family and widespread in applications, has motivated many researchers for design alternative [13]. The fractional order algorithm for the control of dynamic systems has been introduced and superior performance of CRONE (French abbreviation for Commande Robuste d'Ordre Non Entier), over the conventional PID controller, has been successfully demonstrated by Oustaloup [14]. Podlubny [15] has proposed a generalization of the PID controller as $PI^{\lambda}D^{\mu}$ controller which is known as fractional order PID controller, where λ is the non-integer order of integrator and μ is the non-integer order of the differentiator term and demonstrated that the $PI^{\lambda}D^{\mu}$ controller has better response than classical PID controller. Recent studies of $PI^{\lambda}D^{\mu}$ controller are the frequency domain approaches [16], implementations on physical systems [17, 18] and tuning applications [19,20,21].

In this study, a $PI^{\lambda}D^{\mu}$ controller was designed for the active vibration suppression of a smart beam which was modelled as an integer order plant. Designing of $PI^{\lambda}D^{\mu}$ controller was fulfilled in two steps. First, the optimization problem of a regulator controller's required parameters was established by using an experimentally obtained transfer function of the smart beam by using the aforementioned piezoelectric patches as sensor and actuator pairs. In this particular problem, $PI^{\lambda}D^{\mu}$ controller was considered by using a fourth degree approach of continued fraction expansion (CFE) method. Then the optimization problem of finding the $PI^{\lambda}D^{\mu}$ controller's parameters was solved by using both the MATLAB Optimization Toolbox [22] and an optimization model which was developed in house by using MATLAB Simulink package program.

2. Mathematical Model of the Smart Beam

The smart beam (Fig. 1a) was a cantilever aluminium beam (490 x 51 x 2 mm) with eight surface-bonded SensorTech - BM500 (25 x 20 x 0.5 mm) PZT patches [23] (Fig. 1b). A very thin isolation layer was placed between the aluminium beam and PZT patches so that each of the PZT patches may independently be employed as a sensor or an actuator.

(a)

(b)

Fig. 1: (a) Smart beam (b) Piezoelectric patch (SensorTech - BM500 PZT Patch)

Piezoelectric patches were labelled according to their positions on each surface of the aluminium beam (Fig. 2). On surface A, the piezoelectric patches were labelled from 1 to 4 in clockwise direction and on surface B, they were labelled from 1 to 4 in counter-clockwise direction. Piezoelectric patches were identified by number and surface codes, such as piezoelectric patch 1A.

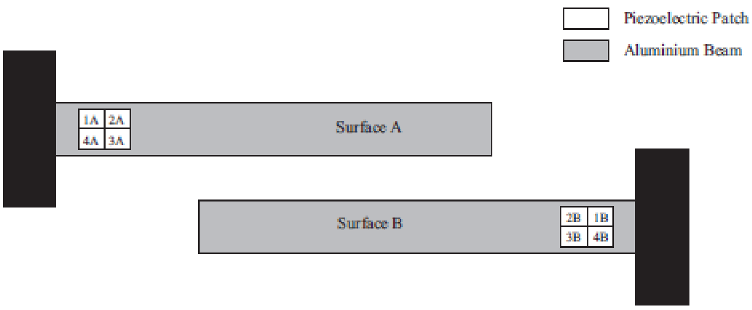

Fig. 2: Piezoelectric patches on smart beam

The experimental frequency response was obtained by simultaneous measurement of the excitation and response signals [24]. The smart beam was excited by four piezoelectric patches which were configured as bimorph to have more actuation ability. These bimorph configured piezoelectric actuator patches were 1A-1B and 4A-4B. During the excitation, the response of the smart beam was monitored by piezoelectric patch 2A. The excitation signal was a swept sine signal from 2 Hz to 48 Hz with 5V peak-to-peak amplitude and generated by HP33120A signal generator. Before transferring this excitation signal to piezoelectric patches, the signal was amplified 30 times by SensorTech SA10 High Voltage Amplifier which uses SensorTech

SA21 High Voltage Power Supply. Bruel and Kjær PULSE 3560C platform was used as the analyzer to obtain the frequency response of the smart beam. The mathematical model of the smart beam was derived by processing the measured frequency response. By using MATLAB’s “fitsys” command located in μ Analysis and Synthesis Toolbox, the required transfer function of the smart beam was obtained. This command was constructed by least square method and the obtained 6th order transfer function was given in Equation (1) [24].

$$G(s)=\frac{0.05513\cdot s^6+0.284\cdot s^5+4067\cdot s^4+1.517\cdot 10^4\cdot s^3+1.586\cdot 10^7\cdot s^2+2.877\cdot 10^7\cdot s+1.567\cdot 10^{10}}{s^6+4.922\cdot s^5+6.611\cdot 10^4\cdot s^4+1.089\cdot 10^5\cdot s^3+2.294\cdot 10^8\cdot s^2+1.815\cdot 10^8\cdot s+2.044\cdot 10^{11}} \quad (1)$$

3. Theory of Fractional Systems

Various definitions and expressions for general fractional integro-differential operation can be found in the literature [16].

The expression for fractional order differentiation is given as [26];

$$ {}_0D_t^{\alpha} y(t) = \frac{1}{\Gamma(1-\gamma)} \int_0^t \frac{y^{(m+1)}(\tau)}{(t-\tau)^{\gamma}} d\tau \quad (2)$$

where $\alpha = m+\gamma$, m is an integer and $0 < \gamma < 1$.

On the other hand, the expression for fractional order integration is defined as [26];

$$ {}_0D_t^{\gamma} = \frac{1}{\Gamma(-\gamma)} \int_0^t \frac{y(\tau)}{(t-\tau)^{1+\gamma}} d\tau \quad , \quad \gamma < 0 \quad (3)$$

Due to the straightforward calculation, Laplace domain is commonly used to express the fractional integro-differential operations. The Laplace transform of fractional order differentiation can be given as;

$$L\left[{}_0D_t^{\alpha} f(t)\right] = s^{\alpha} L[f(t)] - \sum_{k=1}^{n-1} s^k \left[{}_0D_t^{\alpha-k-1} f(t)\right]_{t=0} \quad (4)$$

If the derivatives of the function f(t) are all equal to zero, Equation (4) turns to the following:

$$L\left[{}_0D_t^{\alpha} f(t)\right] = s^{\alpha} L[f(t)] \quad (5)$$

A fractional differential equation for a fractional order control system can be expressed in Equation (6);

$$a_n \frac{d^{\alpha_n} y(t)}{dt^{\alpha_n}} + a_{n-1} \frac{d^{\alpha_{n-1}} y(t)}{dt^{\alpha_{n-1}}} + \cdots\cdots + a_0 \frac{d^{\alpha_0} y(t)}{dt^{\alpha_0}} = b_m \frac{d^{\beta_m} x(t)}{dt^{\beta_m}} + b_{m-1} \frac{d^{\beta_{m-1}} x(t)}{dt^{\beta_{m-1}}} + \cdots\cdots + b_0 \frac{d^{\beta_0} x(t)}{dt^{\beta_0}} \quad (6)$$

where y(t) is the output and x(t) is the input of the system. The Laplace transform of Equation (6) can be obtained in the following form [27].

$$G(s)=\frac{Y(s)}{X(s)}=\frac{b_m s^{\beta_m}+b_{m-1}s^{\beta_{m-1}}+....+b_0 s^{\beta_0}}{a_n s^{\alpha_n}+a_{n-1}s^{\alpha_{n-1}}+.....+a_0 s^{\alpha_0}} \tag{7}$$

where $\alpha_n > \alpha_{n-1} > .... > \alpha_0 \geq 0$ and $\beta_m > \beta_{m-1} > .... > \beta_0 \geq 0$ are satisfied, a_k (k = 0,1,2,...,n) and b_k (k = 0,1,2,...,n) are constants.

The analysis of the Laplace transform and inverse Laplace transform of fractional integro-differential operation in time domain are quite complicated and time consuming. To overcome these drawbacks integer order transfer function is preferred instead of the fractional order transfer function and this can be managed by using the continued fraction expansion (CFE) method [28,29]. Hence, in this study CFE method is used for obtaining the integer order transfer function of the $PI^{\lambda}D^{\mu}$ controller. The fourth order integer approach of a fractional statement given in Equation (8) [29] is selected as it provides better approximation compared to the lower order approaches.

$$s^{\alpha}=\frac{\begin{array}{l}(\alpha^4+10\cdot\alpha^3+35\cdot\alpha^2+50\cdot\alpha+24)\cdot s^4+(-4\cdot\alpha^4-20\cdot\alpha^3+40\cdot\alpha^2+320\cdot\alpha+384)\cdot s^3\\ +(6\cdot\alpha^4-150\cdot\alpha^2+864)\cdot s^2+(-4\cdot\alpha^4+20\cdot\alpha^3+40\cdot\alpha^2-320\cdot\alpha+384)\cdot s+(\alpha^4-10\cdot\alpha^3+35\cdot\alpha^2-50\cdot\alpha+24)\end{array}}{\begin{array}{l}(\alpha^4-10\cdot\alpha^3+35\cdot\alpha^2-50\cdot\alpha+24)\cdot s^4+(-4\cdot\alpha^4+20\cdot\alpha^3+40\cdot\alpha^2-320\cdot\alpha+384)\cdot s^3\\ +(6\cdot\alpha^4-150\cdot\alpha^2+864)\cdot s^2+(-4\cdot\alpha^4-20\cdot\alpha^3+40\cdot\alpha^2+320\cdot\alpha+384)\cdot s+(\alpha^4+10\cdot\alpha^3+35\cdot\alpha^2+50\cdot\alpha+24)\end{array}} \tag{8}$$

4. Design of $PI^{\lambda}D^{\mu}$ Controller

Determination of the $PI^{\lambda}D^{\mu}$ controller parameters was fulfilled in two steps. Firstly, the optimization problem of the regulator controller’s required parameters was established by using the transfer function of the smart beam given in Equation (1). This problem was to design a feedback control which was able to approach the output of the closed loop system to zero. The solution of this problem was actually the minimization of the output of the closed loop system shown in Fig. 3.

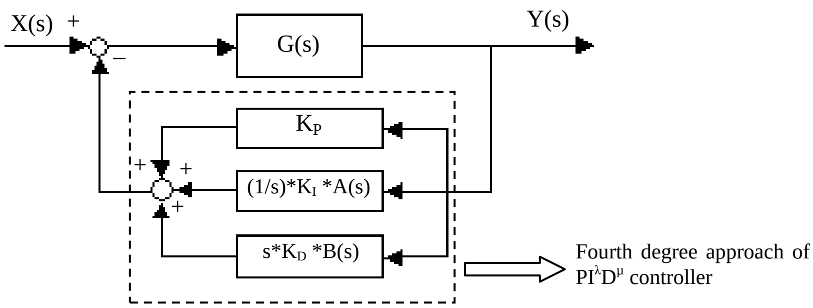

Fig. 3: Closed loop system

Where G(s), X(s), Y(s) in order represented the smart beam, system input, system output transfer functions. K_P, K_I and K_D were the proportionality constant, integral constant and derivative constant respectively. A(s) and B(s) were the fourth degree integer order transfer functions of $1/s^{\lambda}$ and s^{μ} that were obtained by using CFE method, and are given Equation (9) and Equation (10) respectively.

$$\frac{1}{s^{\lambda}} \cong A(s) = \frac{\begin{array}{l}\left(\lambda^4-10\cdot\lambda^3+35\cdot\lambda^2-50\cdot\lambda+24\right)s^4+\left(-4\cdot\lambda^4+20\cdot\lambda^3+40\cdot\lambda^2-320\cdot\lambda+384\right)s^3\\+\left(6\cdot\lambda^4-150\cdot\lambda^2+864\right)s^2+\left(-4\cdot\lambda^4-20\cdot\lambda^3+40\cdot\lambda^2+320\cdot\lambda+384\right)s+\left(\lambda^4+10\cdot\lambda^3+35\cdot\lambda^2+50\cdot\lambda+24\right)\end{array}}{\begin{array}{l}\left(\lambda^4+10\cdot\lambda^3+35\cdot\lambda^2+50\cdot\lambda+24\right)s^4+\left(-4\cdot\lambda^4-20\cdot\lambda^3+40\cdot\lambda^2+320\cdot\lambda+384\right)s^3\\+\left(6\cdot\lambda^4-150\cdot\lambda^2+864\right)s^2+\left(-4\cdot\lambda^4+20\cdot\lambda^3+40\cdot\lambda^2-320\cdot\lambda+384\right)s+\left(\lambda^4-10\cdot\lambda^3+35\cdot\lambda^2-50\cdot\lambda+24\right)\end{array}} \tag{9}$$

$$s^{\mu} \cong B(s) = \frac{\begin{array}{l}\left(\mu^4+10\cdot\mu^3+35\cdot\mu^2+50\cdot\mu+24\right)s^4+\left(-4\cdot\mu^4-20\cdot\mu^3+40\cdot\mu^2+320\cdot\mu+384\right)s^3\\+\left(6\cdot\mu^4-150\cdot\mu^2+864\right)s^2+\left(-4\cdot\mu^4+20\cdot\mu^3+40\cdot\mu^2-320\cdot\mu+384\right)s+\left(\mu^4-10\cdot\mu^3+35\cdot\mu^2-50\cdot\mu+24\right)\end{array}}{\begin{array}{l}\left(\mu^4-10\cdot\mu^3+35\cdot\mu^2-50\cdot\mu+24\right)s^4+\left(-4\cdot\mu^4+20\cdot\mu^3+40\cdot\mu^2-320\cdot\mu+384\right)s^3\\+\left(6\cdot\mu^4-150\cdot\mu^2+864\right)s^2+\left(-4\cdot\mu^4-20\cdot\mu^3+40\cdot\mu^2+320\cdot\mu+384\right)s+\left(\mu^4+10\cdot\mu^3+35\cdot\mu^2+50\cdot\mu+24\right)\end{array}} \tag{10}$$

The next step was the solving of the optimization problem in order to find the $PI^{\lambda}D^{\mu}$ controller's parameters (K_P, K_I, K_D, λ and μ). This was achieved via MATLAB Optimization Toolbox and an optimization model which was developed by using the MATLAB Simulink. Fig. 4 shows the developed MATLAB Simulink optimization model.

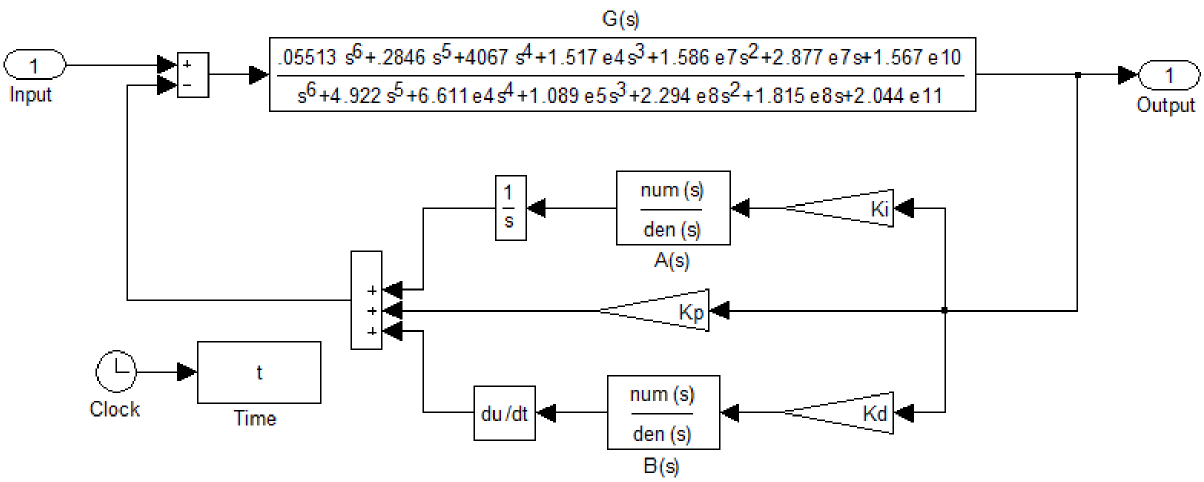

Fig. 4: In-house developed Simulink optimization model

The input signal used in the optimization model is given in Fig. 5.

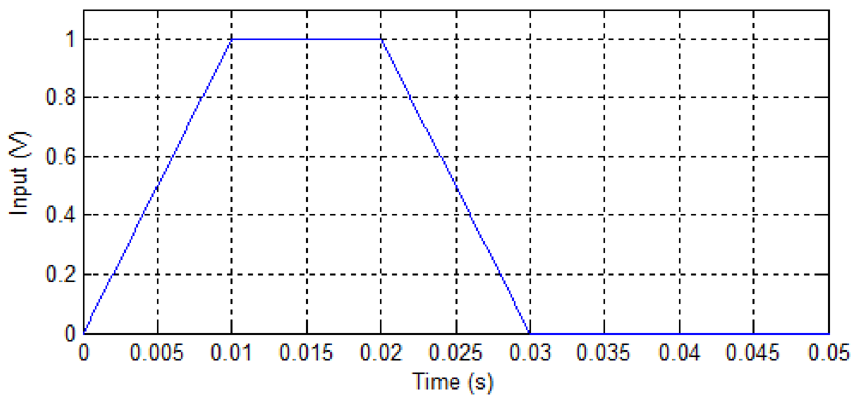

Fig. 5: The input signal used in the optimization model

The impact signal given in Fig. 5 was preferred as an input for the optimization model in the determination of the parameters of $PI^{\lambda}D^{\mu}$ controller. This function resembles the unit impulse function and it's Laplace transform results in unity. The response of the closed loop system was minimised by using the MATLAB Optimisation Toolbox command, "lsqnonlin", and for the given problem the following optimum values were determined. K_P= 0.6300, K_I=0.0504, K_D=1.9688, λ=0.9788 and μ=1.3143.

5. Simulations Studies

The time domain simulations were performed for both the open and the closed loop systems. The PZTs were utilized as either sensor or actuator depending on their locations. During the simulations, the input voltage is applied to the PZT actuators and the output voltage due to the PZT sensor is evaluated.

5.1 Free Vibrations of the Smart Beam

In order to obtain the free vibration response, a tip displacement corresponding to 1V was applied on sensor patch. The open and the closed loop system responses of the smart beam to this particular input are presented in Fig. 6. The absolute maximum amplitudes are 0.1333 V and 0.0128 V for the open loop and the closed loop systems respectively.

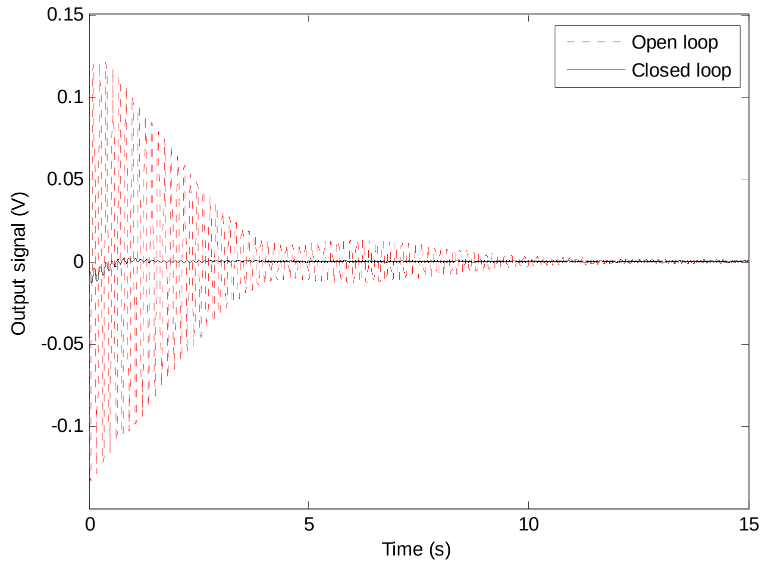

Fig. 6: Free vibration response of the smart beam

5.2 Suppression of Forced Vibrations of the Smart Beam

The forced vibration simulations were performed by using four different input signals. Those signals were harmonic and transient signals as well. The first input was the impact signal given in Fig. 5 and it's passive and closed loop transient state responses are presented in Fig. 7.

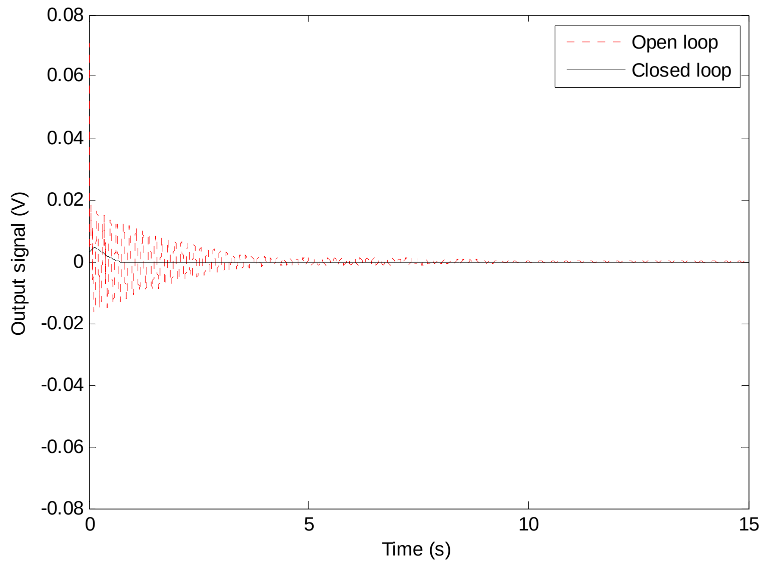

Fig. 7: The response of the smart beam to transient input

It can be seen from Fig. 7 that absolute maximum amplitude of the open loop response is 0.0705 V and that of closed loop one is 0.0044 V. The controller accomplished in suppressing the vibration levels within approximately three seconds.

The next two input signals were aimed to excite the beam at the first and the second resonance frequencies. Those frequencies were 6.69 Hz and 39.78 Hz. The sinusoidal disturbance signals had the magnitude of 1V. The open and closed loop responses of the smart beam at the first (ω_1) and the second (ω_2) resonance frequencies are shown in Fig. 8 and Fig. 9 respectively. In these figures, the controller was switched on at the fifth second of the simulations.

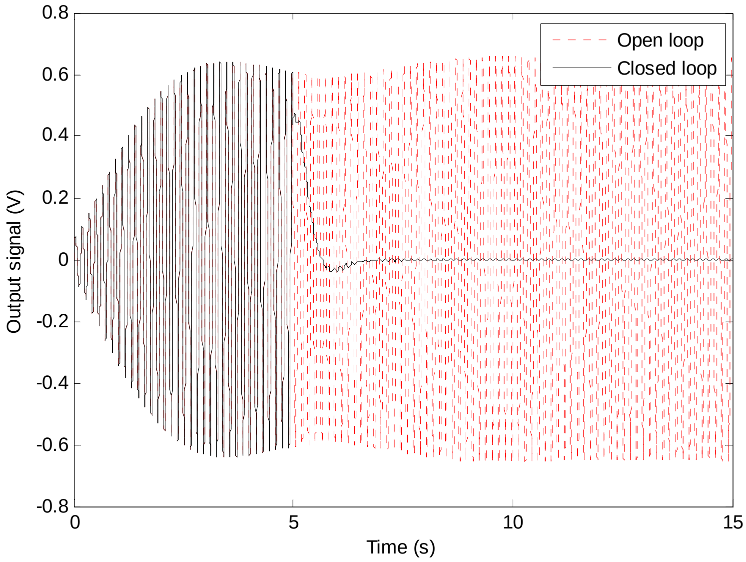

Fig. 8: Forced response at ω_1

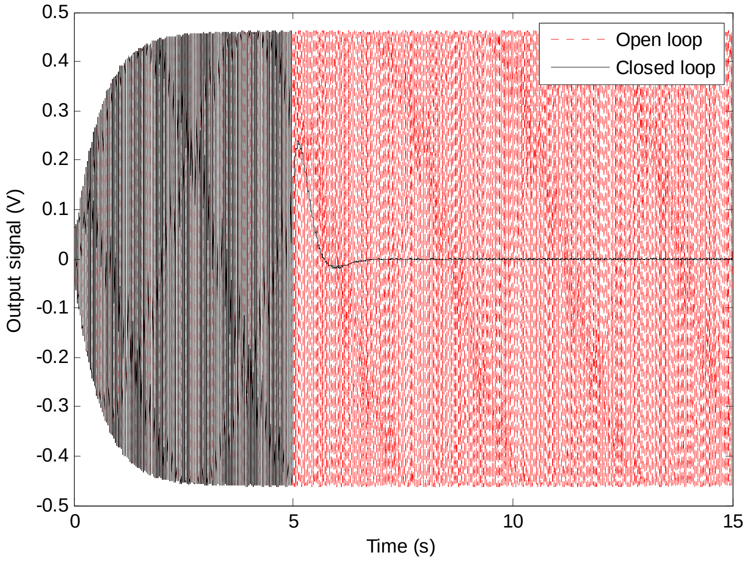

Fig. 9: Forced response at ω_2

In Fig.8, which corresponded to the harmonic excitation at first resonance frequency; absolute maximum amplitude of open loop case is 0.6580 V and that of closed loop one is 0.0086 V. When the responses at the second resonance frequency were considered in Fig. 9, the absolute maximum amplitudes of open and closed loop cases are 0.4586 V and 0.0014 V respectively.

Finally, a unit step input was applied to the system in order to show the performance of the controller for continuous input. Fig. 10 presents the step response of the open and the closed loop systems and it is clearly seen that the controller is successful in suppressing of closed loop output. Fig. 11, on the other hand, shows the output voltage of the $PI^{\lambda}D^{\mu}$ controller for the same input. It can bee seen from this figure that the output voltage of the $PI^{\lambda}D^{\mu}$ controller is approaching to the input signal with time by achieving the design goal of the controller.

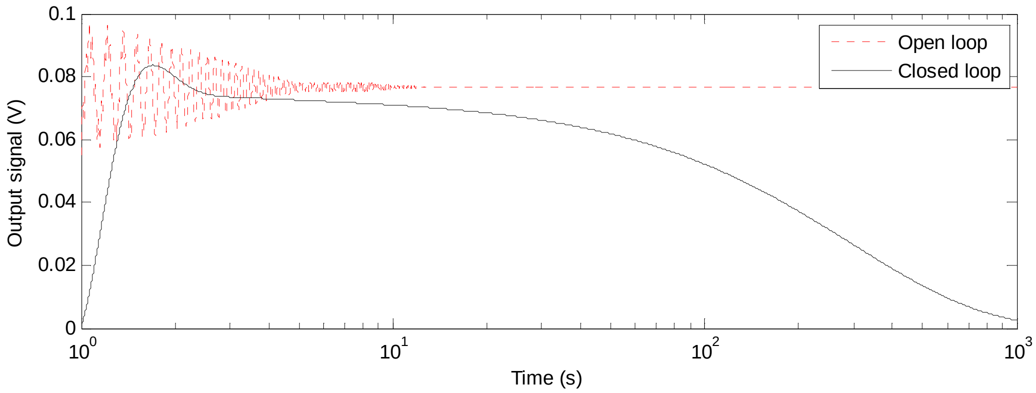

Fig. 10: Step response

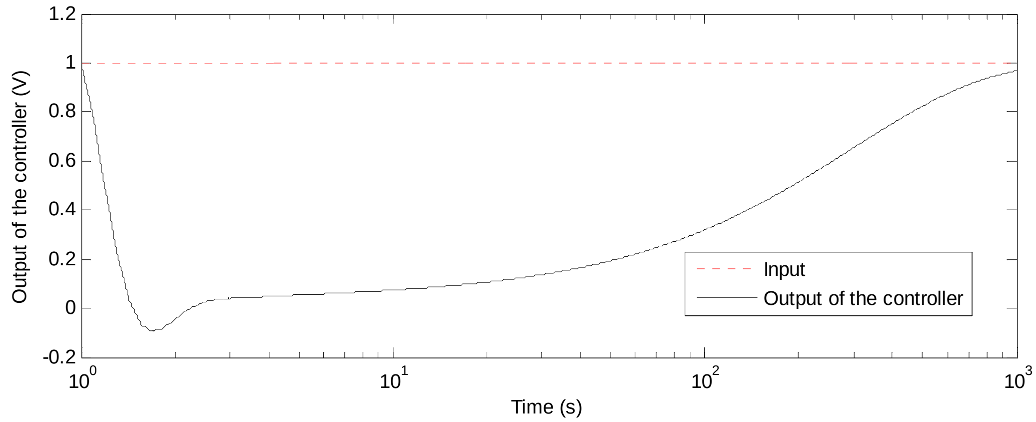

Fig. 11: Output voltage of the $PI^{\lambda}D^{\mu}$ controller for step input

6. Conclusion

In this study, a $PI^{\lambda}D^{\mu}$ controller was designed for the active vibration suppression of a smart beam equipped with PZT (Lead Zirconate Titanate) patches that was modelled as an integer order plant. Time domain simulations which were done for different inputs have demonstrated the effectiveness of the $PI^{\lambda}D^{\mu}$ controller in suppressing of the vibration levels of the smart beam. In the future the frequency domain simulations will also be conducted and robust $PI^{\lambda}D^{\mu}$ controller and gain scheduling $PI^{\lambda}D^{\mu}$ controller will be studied.

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