

# Active Vibration Suppression of a Smart Beam by Using a Fractional Control

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## ABSTRACT

In this study, a fractional order controller was designed and experimentally applied for the active vibration suppression of a smart beam. The smart beam was a cantilever aluminium beam equipped with eight symmetrically located surface-bonded PZT (Lead-Zirconate-Titanate) patches which were used both as sensor and actuator. For this particular application, a group of PZT patches closed to the root of the beam was used as actuators in the bimorph configuration and a single patch was nominated as a sensor. Fractional order controllers were known to provide better flexibility in adjusting the gain and phase characteristics than their integer-order counterparts. In the design of the controller, first, the fractional order differential effect was considered by using a fourth degree approach of continued fraction expansion (CFE) method. Then a filter was designed to characterize the dynamic properties of the smart beam in the first flexural mode. Finally, the controller was obtained by filtering the aforementioned differential effect. In order to evaluate the closed-loop frequency domain performances, the simulations were performed on various fractional orders of the differential effect and a controller was then selected for the experimental verifications. The obtained time domain responses have shown that the fractional order controller successfully suppressed the vibration levels of the first flexural mode of the smart beam.

**Keywords:** *smart beam, PZT, vibration, fractional control*

## 1 Introduction

Smart structures are composed of passive structures and smart materials/elements. The smart structures can sense an external disturbance and consequently in real time respond to that. These structures are widely used in Aerospace Engineering, particularly in active vibration control applications [1],

Fractional order control systems have transfer functions with fractional derivatives and fractional integrals. It is not easy to compute the frequency and time domain behaviors of such fractional order transfer functions with available commercial software packages, since those programs have been prepared to deal with the integer powers only [2]. Hence, the problem of integer order approximations of fractional order functions deserves further attention. A fractional transfer function can be replaced with an integer order transfer function which has almost the same behavior with the actual transfer function but much more easy to deal with [3,4].

There are several methods for obtaining the integer approximations of fractional order systems. Carlson's method, Matsuda's method, Oustaloup's method, the Grünwald-Letnikov approximation,

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Maclaurin series based approximations and time response based approximations are a few to mention. [5]. One of the most important approximations for fractional order systems is the Continued Fraction Expansion (CFE) method [6].

In this study, a fractional controller was designed for the active vibration suppression of a smart beam which was modelled as an integer order plant. In the design of the controller, first, the fractional order differential effect was considered by using a fourth degree approach of continued fraction expansion method. Then a filter was designed to characterize the dynamic properties of the smart beam in its first flexural mode. Finally, the controller was obtained by filtering the aforementioned differential effect. The developed controller was verified through a variety of experiments conducted in time domain and frequency domain.

## 2 Smart Beam and its Experimental Identification

The smart beam given in Figure 1 is a cantilever aluminium beam having the dimensions of 490 x 51 x 2 mm and eight surface bonded SensorTech - BM500 (25 x 20 x 0.5 mm) PZT (Lead - Zirconate -Titanate) patches [7]. A thin isolation layer is placed between the aluminium beam and PZT patches, so that each PZT patch may be employed as a sensor and an actuator independently.

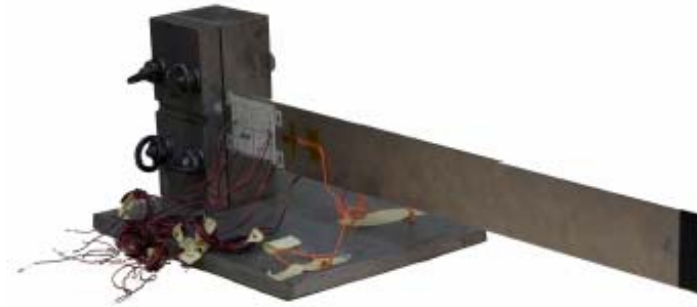


Figure 1: Smart beam used in the study

In the study, the system model of the smart beam was experimentally determined as a 6<sup>th</sup> order transfer function and is given in Equation 1 as [8]:

$$G(s) = \frac{0.06449 \cdot s^6 + 13.42 \cdot s^5 + 288.7 \cdot s^4 + 54660 \cdot s^3 + 3.548 \cdot 10^5 \cdot s^2 + 5.55 \cdot 10^7 \cdot s + 7.102 \cdot 10^7}{s^6 + 191.6 \cdot s^5 + 6085 \cdot s^4 + 741800 \cdot s^3 + 1.211 \cdot 10^7 \cdot s^2 + 7.179 \cdot 10^8 \cdot s + 7.89 \cdot 10^9} \quad (1)$$

Figure 2 shows the comparison of the experimentally measured and analytically estimated transfer functions of the smart beam and indicates the accuracy of the developed smart beam model in terms of both the magnitude and the phase.

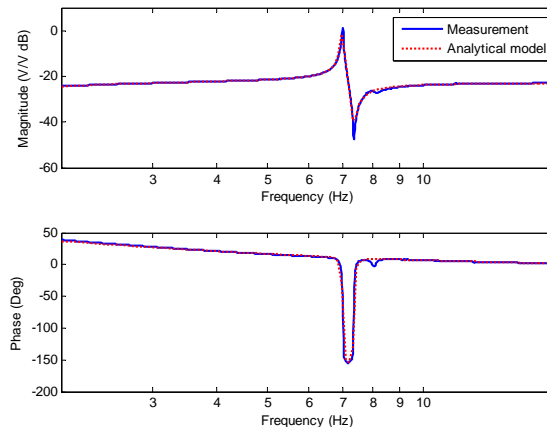


Figure 2: Experimentally measured and analytically estimated transfer functions of the smart beam

### 3 Fractional Controller Design

Development of an active controller for a system is analogous to the determination of a suitable damping ratio for the same system. Since the viscous damping force is proportional with the velocity and the velocity is the time rate of change of the displacement, the knowledge of the differential effect becomes important for the controller design. In this study the differential effect was included as the fractional one.

The controller for active vibration suppression was synthesized in two steps. First, the fractional differential effect (fractional velocity signal) of the smart beam was derived from the measurement signal by using the fractional derivative effect  $s^\mu$ . The  $s^\mu$  was considered by using a fourth degree approach of CFE method and is given in Equation 2.

$$s^\mu \cong \frac{(\mu^4 + 10 \cdot \mu^3 + 35 \cdot \mu^2 + 50 \cdot \mu + 24) \cdot s^4 + (-4 \cdot \mu^4 - 20 \cdot \mu^3 + 40 \cdot \mu^2 + 320 \cdot \mu + 384) \cdot s^3 + (6 \cdot \mu^4 - 150 \cdot \mu^2 + 864) \cdot s^2 + (-4 \cdot \mu^4 + 20 \cdot \mu^3 + 40 \cdot \mu^2 - 320 \cdot \mu + 384) \cdot s + (\mu^4 - 10 \cdot \mu^3 + 35 \cdot \mu^2 - 50 \cdot \mu + 24)}{(\mu^4 - 10 \cdot \mu^3 + 35 \cdot \mu^2 - 50 \cdot \mu + 24) \cdot s^4 + (-4 \cdot \mu^4 + 20 \cdot \mu^3 + 40 \cdot \mu^2 - 320 \cdot \mu + 384) \cdot s^3 + (6 \cdot \mu^4 - 150 \cdot \mu^2 + 864) \cdot s^2 + (-4 \cdot \mu^4 - 20 \cdot \mu^3 + 40 \cdot \mu^2 + 320 \cdot \mu + 384) \cdot s + (\mu^4 + 10 \cdot \mu^3 + 35 \cdot \mu^2 + 50 \cdot \mu + 24)} \quad (2)$$

Then a filter was designed to characterize the dynamic properties of the smart beam in the first flexural mode and the controller was obtained by passing the above differential effect from the filter  $H(s)$ . The filter  $H(s)$  is given in Equation 3.

$$H(s) = \frac{1}{s^2 + 8.796 \cdot s + 1934} \quad (3)$$

The block diagram of the closed loop system is given Figure 3.  $X(s)$ ,  $Y(s)$  and  $K$  stand for the system input, the system output and the controller gain in Laplace domain in order.

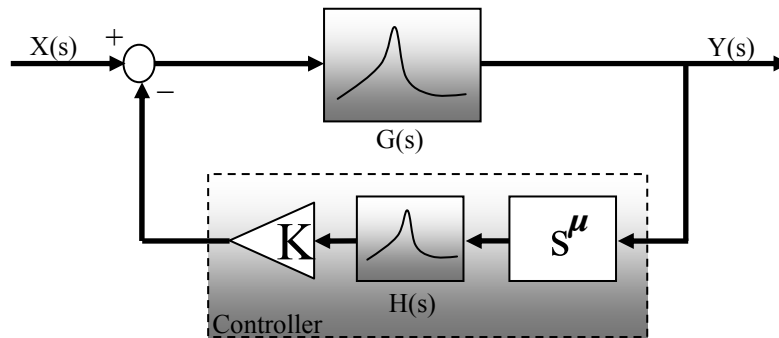


Figure 3: Block diagram of the developed fractional controller

### 4 Simulation Results of the Smart Beam

In Figure 4, the frequency responses of the open loop system and closed loop systems are given for different values of the fractional order  $\mu$ . The controller gain was constant as  $K=100$ . It can be seen from Figure 4 that increase in the fractional order provides more effective suppression in the resonance region of the open loop frequency response. However, this increase causes a shift at the resonance frequency and a performance loss at low frequency region which was also accompanied. This effect is more prominent for higher values of the fractional order  $\mu$ . This can better be explained with the help of Figure 5 which shows the pole-zero map of the closed loop responses. As it can be seen from Figure 5 that there are right half plain poles for  $\mu=0.8$  and  $\mu=0.9$  cases. That is likely to

make these cases as unstable. Hence, a value of the fractional order was selected to yield comparatively better performances both at the resonance and also at the off-resonant region as  $\mu=0.2$ .

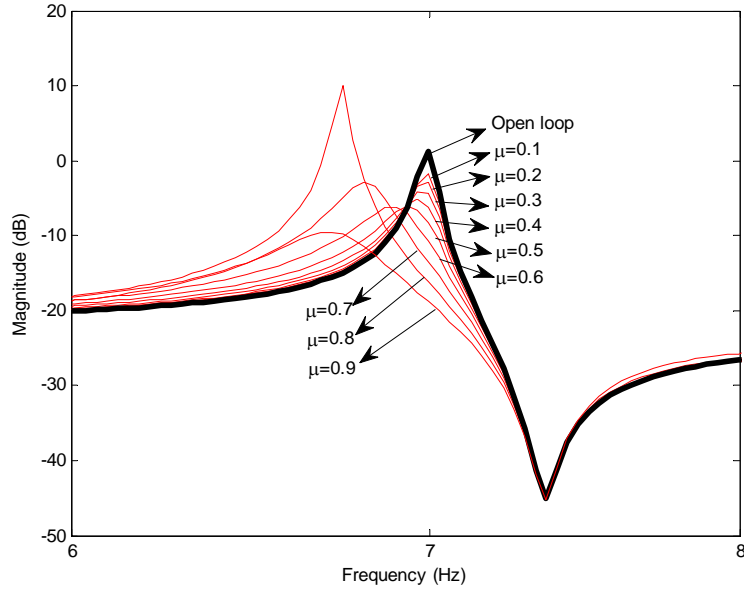


Figure 4: Frequency response of the smart beam for different values of the fractional order  $\mu$

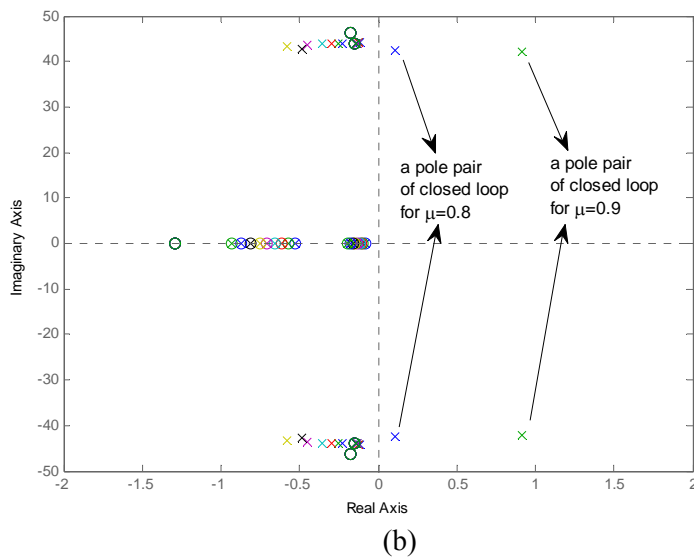
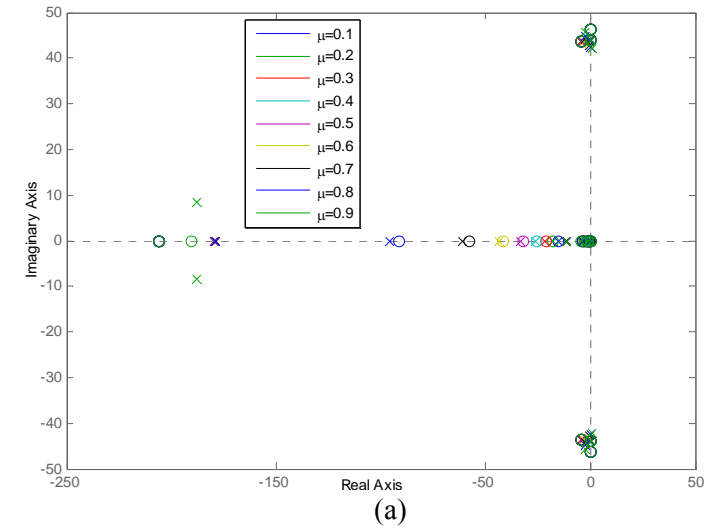


Figure 5: (a) Pole-zero map of the closed loop responses  
(b) Zoomed pole-zero map of the closed loop responses

The control design was further conducted by considering five different values of control gain  $K$  (100, 200, 300, 400 and 500). Figure 6.a shows the simulated closed loop frequency responses for different  $K$  values together with the open loop frequency response of the smart beam. Figure 6.b further expands the response around the resonance value of approximately 7 Hz. It can be seen that increasing the  $K$  increases the effectiveness of the controller in resonance region. In off-resonant region and especially at lower frequencies increasing  $K$ , although slightly, decreases the controller effectiveness.

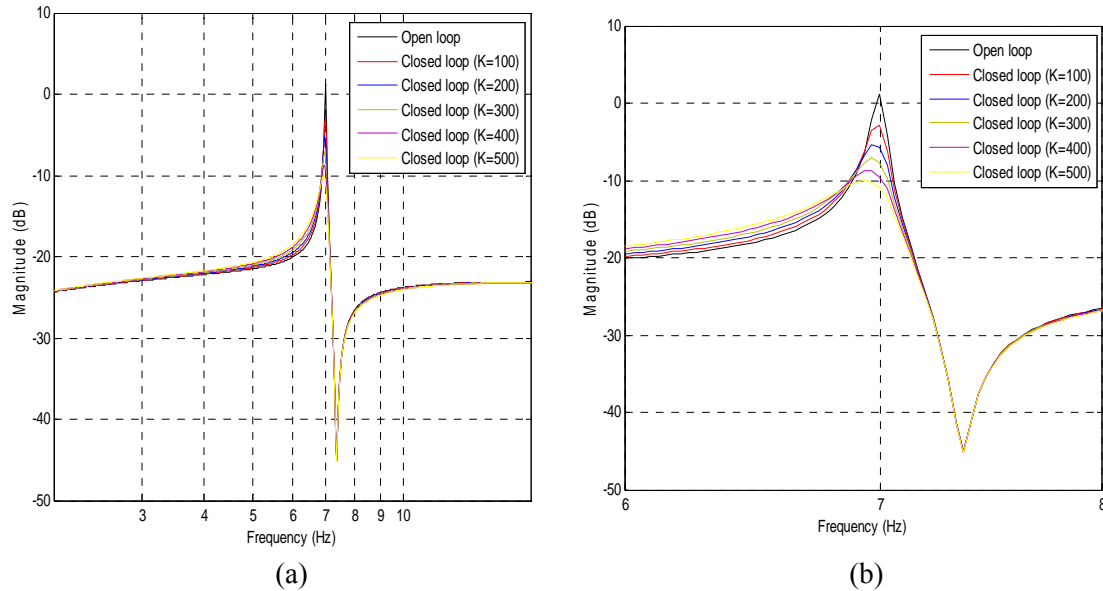


Figure 6: (a) Response of the smart beam for different values of the control gain  $K$  (b) Expanded response of the smart beam for different values of the control gain  $K$

Figure 7 shows the simulated closed loop step responses for different  $K$  values together with the open loop step response of the smart beam. The rms values of the step responses of the open loop system and closed loop systems are given in Table 1. From Figure 7, it can be seen that increasing the  $K$  increases the effectiveness of the controller by decreasing of the vibration settling time. But Table 1 further shows that increasing the controller gain above  $K=300$  is not so effective on the closed loop response.

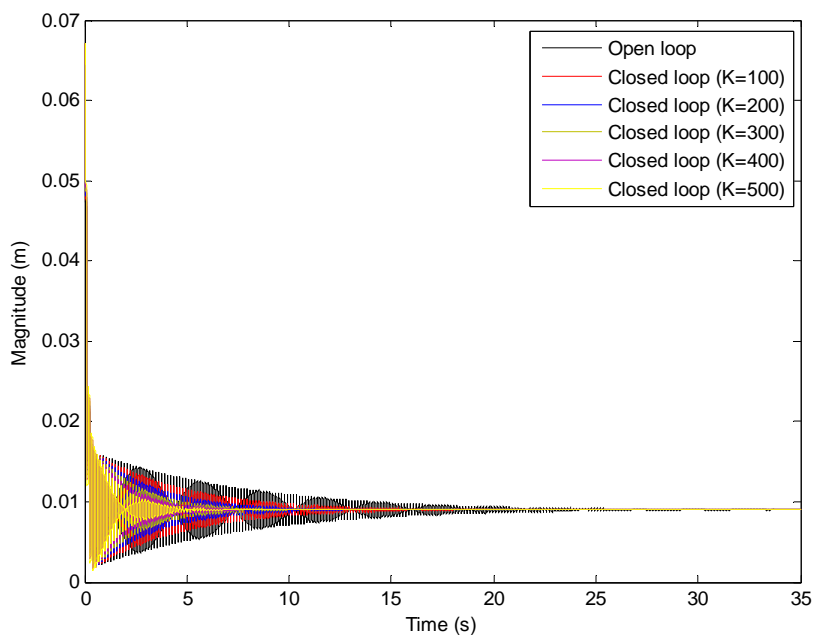


Figure 7: Simulated step response of the smart beam

Table 1: RMS values of the simulated step responses of the smart beam

	Open loop	K=100	K=200	K=300	K=400	K=500
RMS value	1.8100	1.8013	1.7980	1.7967	1.7964	1.7967

## 5 Experimental Results of the Smart Beam

The smart beam was given an initial 8 mm tip deflection and the ensuing motion was measured for open and closed loop time responses. The results are presented in Figure 8 and it shows the controlled time responses of the smart beam for K=100, K=200, K=300, K=400 and K=500 cases. The settling times were determined to be nearly 6.6, 4.6, 3.9, 3.7 and 3.6 seconds for increasing K values in order. Hence, the designed controller for K=500 proves to be the most effective on suppressing the free vibrations of the smart beam though no significant response difference was observed between the K=400 and K=500 cases.

Then, the smart beam was excited at its first resonance frequency by the help of PZT patches. The effects of controllers on the suppression of the forced vibrations are shown in Figure 9. For these cases, the suppression rates, which is defined in Equation 4 at the first resonance frequency, were calculated approximately as 63, 79, 86, 90 and 91 percent, for K=100, K=200, K=300, K=400 and K=500 cases in order.

$$\text{Suppression Rate} = \frac{[\text{Open Loop Magnitude}]_{\text{MAX}} - [\text{Closed Loop Magnitude}]_{\text{MAX}}}{[\text{Open Loop Magnitude}]_{\text{MAX}}} \times 100 \quad (4)$$

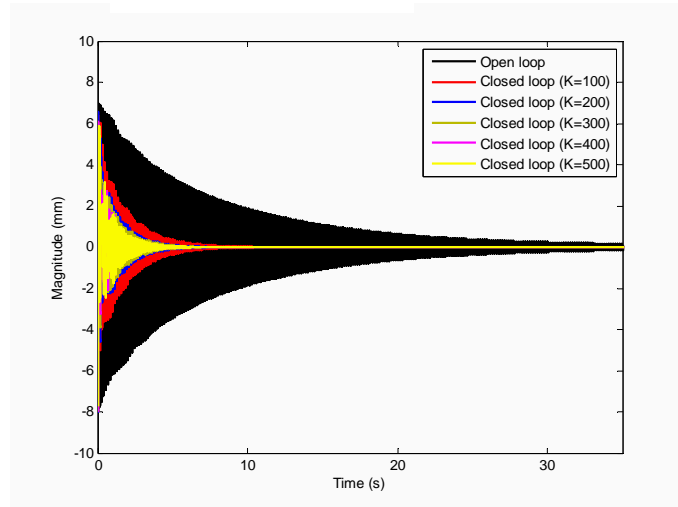


Figure 8: Experimental free vibration responses of the smart beam

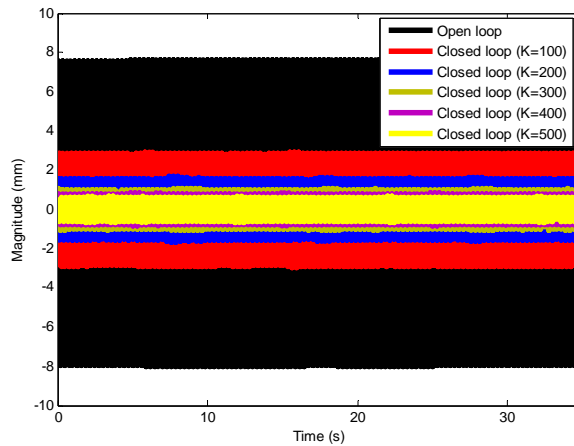


Figure 9: Experimental forced vibration responses of the smart beam at its first resonance frequency

The experimentally obtained open and closed loop frequency response curves of the smart beam are given in Figure 10. It has been determined that the controller with  $K=200$  has shifted the system resonance frequency to 6 Hz,  $K=300$  controller to 5.75 Hz,  $K=400$  controller to 5.5 Hz and  $K=500$  controller to 5.375 Hz. Furthermore the controllers with and above  $K=200$  yielded erroneous results by leading to the classical water bed effect at the low frequency off-resonant region.

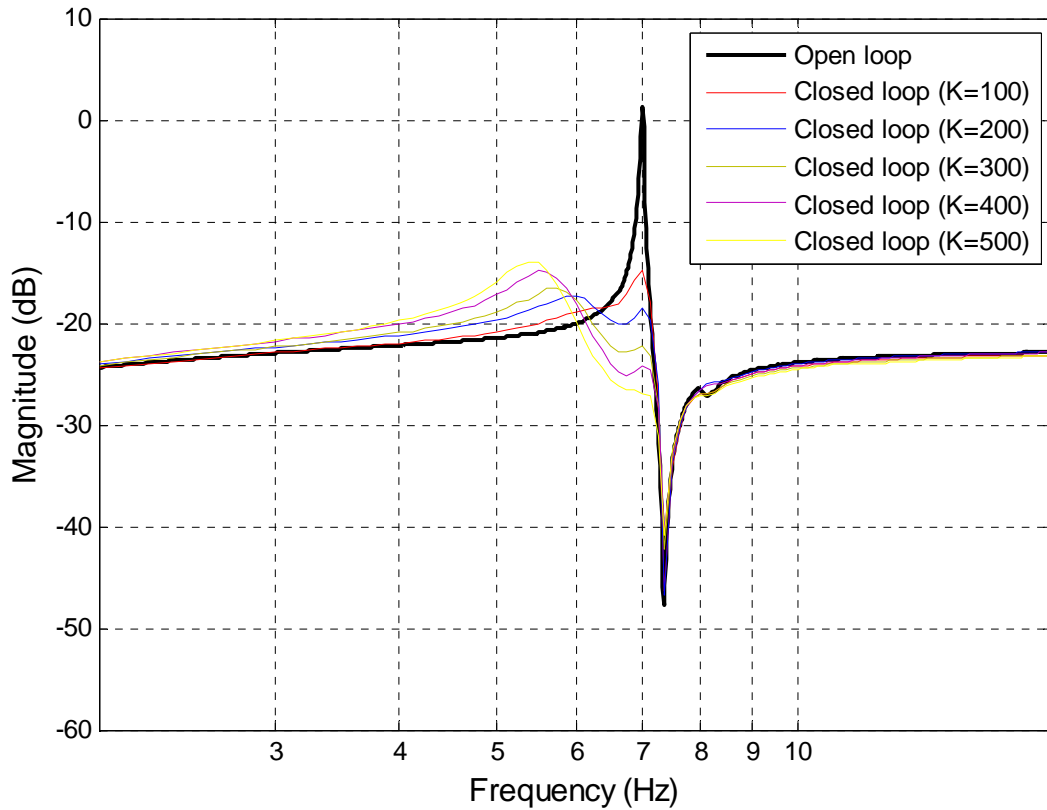


Figure 10: Open and closed loop experimental frequency responses of the smart beam

The vibration levels of the controllers with different  $K$  values were determined and are given in Table 2. Considering the behavior at the resonant region and also at the off-resonant region it can be concluded that, within the limits of the current study, the best performance was shown by  $K=100$  case.

Table 2: Vibration levels for different controllers at the shifted resonance frequencies

Control cases	Resonance frequency was shifted to	Vibration level at the shifted resonance frequency
$K=100$	7.0 Hz	-14.8 dB
$K=200$	6.0 Hz	-17.3 dB
$K=300$	5.75 Hz	-16.5 dB
$K=400$	5.5 Hz	-14.8 dB
$K=500$	5.375 Hz	-13.9 dB

## 6 Conclusion

The design and implementation of a fractional controller which was considered by using CFE method was presented for suppressing the first flexural resonance level of a smart beam. The design was fulfilled for five different values of the controller gain. Increase in the controller gain value provided an improvement in the free response and forced response of the smart beam. However this caused a shift in the resonance frequency and resulted in performance loss at low frequency off-resonant region indicating classical water bed effect between the controller gain and performance.

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