Performance analysis of a fractional controller developed for the vibration suppression of a smart beam

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ABSTRACT: Fractional order control systems have transfer functions with fractional derivative and fractional integrals. Since the vast portion of the currently available electronic hardware is suitable for integer order input, there are several methods for obtaining integer approximations of fractional order systems. In this study a fractional order controller developed via Continued Fraction Expansion (CFE) method, was designed and implemented for the suppression of the flexural vibrations of a smart beam. The first, second, third and fourth order approximations of the CFE method were studied for the performance analysis of the controller. The robustness analysis of the developed controllers was also conducted by attaching point masses at the free end of the smart beam. Experimentally obtained free and forced vibration results demonstrated that the increase in the order yielded better performance around the first flexural resonance region of the smart beam and also increased the robustness of the closed loop controller.

1 INTRODUCTION

In order to suppress the excessive level of vibrations of the aerospace structures, the active vibration control methods are widely applied by scientists and engineers. The atomic lattice structure property of piezoelectric materials enables them to be used as actuator and/or as sensor in active vibration control. In the research studies previously conducted in the Department of Aerospace Engineering of METU, the PZT patches were used for the active vibration suppression of cantilever beam-like and plate-like structures (Sahin 2008, Onat 2001).

Fractional order control systems have transfer functions comprising fractional derivatives s^{α} and fractional integrals $s^{-\alpha}$ where $\alpha \in R$.These fractional orders must be approximated to integer values and there are many methods for doing that (Charef 2006, Chen 2004, Dorcak 2003, Krishna 2008, Podlubny 2002, Shyu 2009, Varshney 2007, Vinagre 2001). One of the most important approximations for fractional order systems is the CFE method (Krishna 2008).

In this study a fractional order controller developed by using the CFE method was designed and implemented for the suppression of the flexural vibrations of a smart beam. The first, second, third and fourth order approximations of CFE method were studied for the performance analysis of the controller. The robustness analysis of the controllers was also conducted by attaching point masses to the free end of the smart beam. Experimentally obtained results were presented for the suppression of the free and forced vibrations.

2 SMART BEAM AND ITS EXPERIMENTAL IDENTIFICATION

The smart beam given in Figure 1 is a cantilever aluminium beam having the dimensions of 490 x 51 x 2 mm and eight surface bonded SensorTech - BM500 (25 x 20 x 0.5 mm) PZT patches (Sensor, 2002). A thin isolation layer is placed between the aluminium beam and PZT patches, so that each PZT patch may be employed as a sensor and an actuator independently.

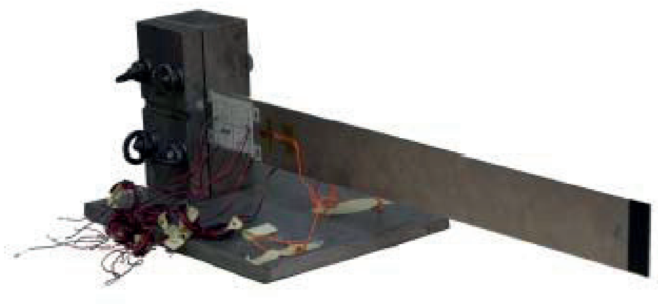


Figure 1. Smart beam used in the study.

In the study, the system model of the smart beam was experimentally determined as (Onat 2011a):

$$G(s)=\frac{0.06449\cdot s^6+13.42\cdot s^5+288.7\cdot s^4+54660\cdot s^3+3.548\cdot 10^5\cdot s^2+5.55\cdot 10^7\cdot s+7.102\cdot 10^7}{s^6+191.6\cdot s^5+6085\cdot s^4+741800\cdot s^3+1.211\cdot 10^7\cdot s^2+7.179\cdot 10^8\cdot s+7.89\cdot 10^9} \quad (1)$$

Figure 2 shows the comparison of the experimentally measured and analytically estimated transfer functions of the smart beam and indicates the accuracy of the developed smart beam model in terms of both the magnitude and the phase.

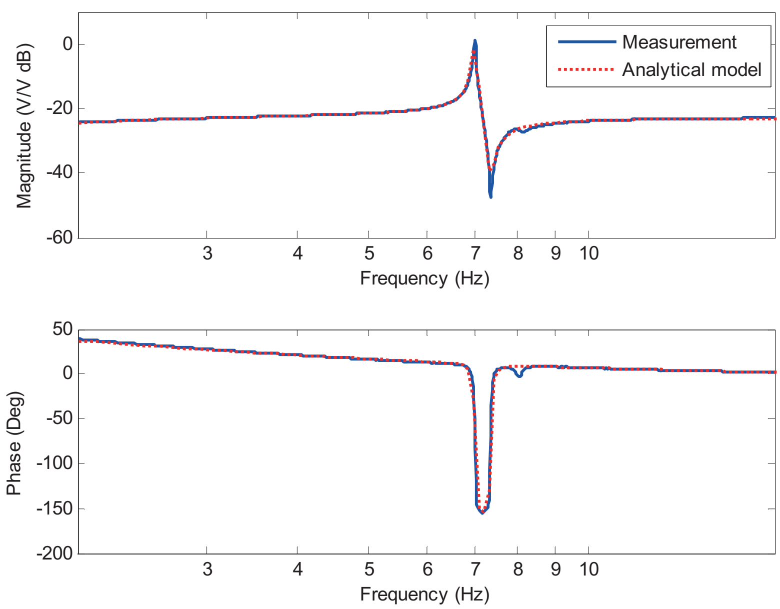


Figure 2. Experimentally measured and analytically estimated transfer functions of the smart beam.

3 FRACTIONAL CONTROLLER DESIGN

In this study the differential effect was included as the fractional one and the controller for active vibration suppression was synthesized in two steps. First, the fractional differential effect of the smart beam was derived from the measurement signal by using the fractional derivative effect s^{μ}. In this study, various approximations for s^{μ} were considered by using first, second, third and fourth degree approach of CFE method. These approximations are given in Equations 2 to 5 in ascending order (Ozyetkin 2010).

$$s^{\mu} \cong \frac{(1+\mu)\cdot s+(1-\mu)}{(1-\mu)\cdot s+(1+\mu)} \tag{2}$$

$$s^{\mu} \cong \frac{\left(\mu^2+3\cdot\mu+2\right)\cdot s^2+\left(-2\cdot\mu^2+8\right)\cdot s+\left(\mu^2-3\cdot\mu+2\right)}{\left(\mu^2-3\cdot\mu+2\right)\cdot s^2+\left(-2\cdot\mu^2+8\right)\cdot s+\left(\mu^2+3\cdot\mu+2\right)} \tag{3}$$

$$s^{\mu} \cong \frac{\begin{array}{l}\left(\mu^3+6\cdot\mu^2+11\cdot\mu+6\right)\cdot s^3+\left(-3\cdot\mu^3-6\cdot\mu^2+27\cdot\mu+54\right)\cdot s^2\\ +\left(3\cdot\mu^3-6\cdot\mu^2-27\cdot\mu+54\right)\cdot s+\left(-\mu^3+6\cdot\mu^2-11\cdot\mu+6\right)\end{array}}{\begin{array}{l}\left(-\mu^3+6\cdot\mu^2-11\cdot\mu+6\right)\cdot s^3+\left(3\cdot\mu^3-6\cdot\mu^2-27\cdot\mu+54\right)\cdot s^2\\ +\left(-3\cdot\mu^3-6\cdot\mu^2+27\cdot\mu+54\right)\cdot s+\left(\mu^3+6\cdot\mu^2+11\cdot\mu+6\right)\end{array}} \tag{4}$$

$$s^{\mu} \cong \frac{\begin{array}{l}\left(\mu^4+10\cdot\mu^3+35\cdot\mu^2+50\cdot\mu+24\right)\cdot s^4+\left(-4\cdot\mu^4-20\cdot\mu^3+40\cdot\mu^2+320\cdot\mu+384\right)\cdot s^3\\ +\left(6\cdot\mu^4-150\cdot\mu^2+864\right)\cdot s^2+\left(-4\cdot\mu^4+20\cdot\mu^3+40\cdot\mu^2-320\cdot\mu+384\right)\cdot s\\ +\left(\mu^4-10\cdot\mu^3+35\cdot\mu^2-50\cdot\mu+24\right)\end{array}}{\begin{array}{l}\left(\mu^4-10\cdot\mu^3+35\cdot\mu^2-50\cdot\mu+24\right)\cdot s^4+\left(-4\cdot\mu^4+20\cdot\mu^3+40\cdot\mu^2-320\cdot\mu+384\right)\cdot s^3\\ +\left(6\cdot\mu^4-150\cdot\mu^2+864\right)\cdot s^2+\left(-4\cdot\mu^4-20\cdot\mu^3+40\cdot\mu^2+320\cdot\mu+384\right)\cdot s\\ +\left(\mu^4+10\cdot\mu^3+35\cdot\mu^2+50\cdot\mu+24\right)\end{array}} \tag{5}$$

Then a filter was designed in order to represent the dynamical characteristics of the first flexural mode of the smart beam. The filter, H(s), is given in Equation 6.

$$H(s)=\frac{1}{s^2+8.554\cdot s+1829} \tag{6}$$

The block diagram of the closed loop system is given Figure 3. X(s), Y(s) and K stand for the system input, the system output and the controller gain in Laplace domain in order.

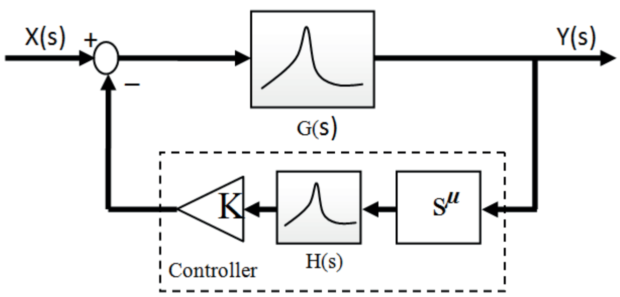
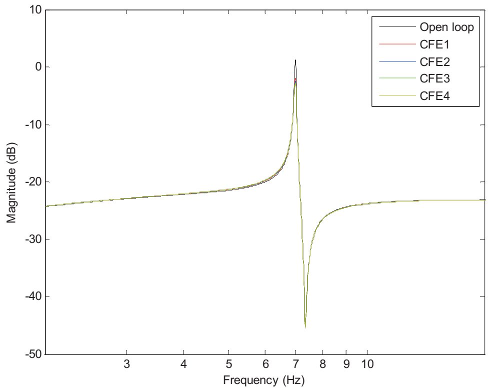


Figure 3. Block diagram of the developed fractional controller.

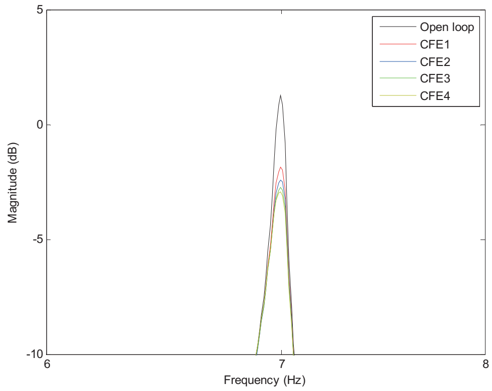
4 SIMULATION RESULTS OF THE SMART BEAM

The gain of the controller was kept constant at K=100 and μ= 0.2 was selected (Onat 2011b). The designed fractional controller was studied by considering four different CFE approximations as the first, second, third and fourth approximations. The closed loop system which was formed by using the first order approximation of the CFE method was named as CFE1. Other closed loops were named accordingly in ascending order.

Figure 4a shows the simulated closed loop frequency responses for different approximation orders of the CFE method together with the open loop frequency response of the smart beam. Figure 4b further expands the response around the resonance value of approximately 7 Hz. It can be seen that increasing the approximation orders increases the effectiveness of the controller in resonance region.



(a)



(b)

Figure 4. (a) Response of the smart beam for different order approximations of the CFE method (b) Zoomed response of the smart beam for different order approximation of the CFE method.

5 EXPERIMENTAL RESULTS OF THE SMART BEAM

In each case, the smart beam was given an initial 8 mm tip deflection and the ensuing motion was measured for open and closed loop time responses. The results are presented in Figure 5. The settling times were recorded to be nearly 19.5, 12.7, 4.2 and 8.1 seconds for CFE1, CFE2, CFE3 and CFE4 cases respectively.

Then, the smart beam was excited at its first resonance frequency (approximately at 7 Hz) by the help of PZT patches. The effects of controllers on the suppression of the forced vibrations are shown in Figure 6. For these cases, the suppression rate at the first resonance frequency, which is given in Equation 7, was calculated approximately as 34.4, 64.5, 87.5 and 78.5 percent, for CFE1, CFE2, CFE3 and CFE4 cases respectively.

$$\text{Suppression Rate} = \frac{[\text{Open Loop Magnitude}]_{MAX} - [\text{Closed Loop Magnitude}]_{MAX}}{[\text{Open Loop Magnitude}]_{MAX}} \times 100 \qquad (7)$$

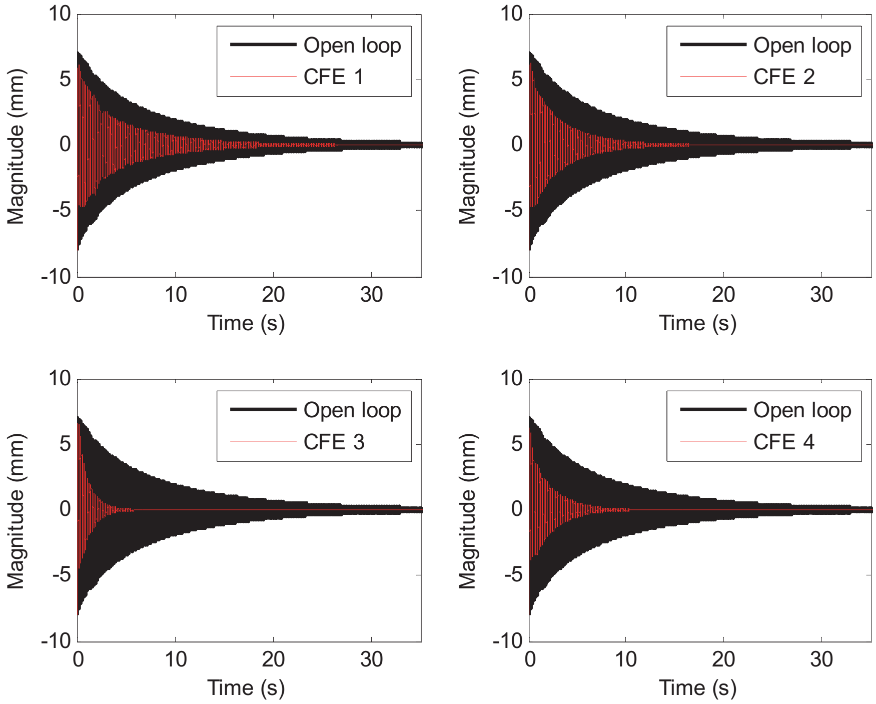


Figure 5. Experimental free vibration responses of the smart beam.

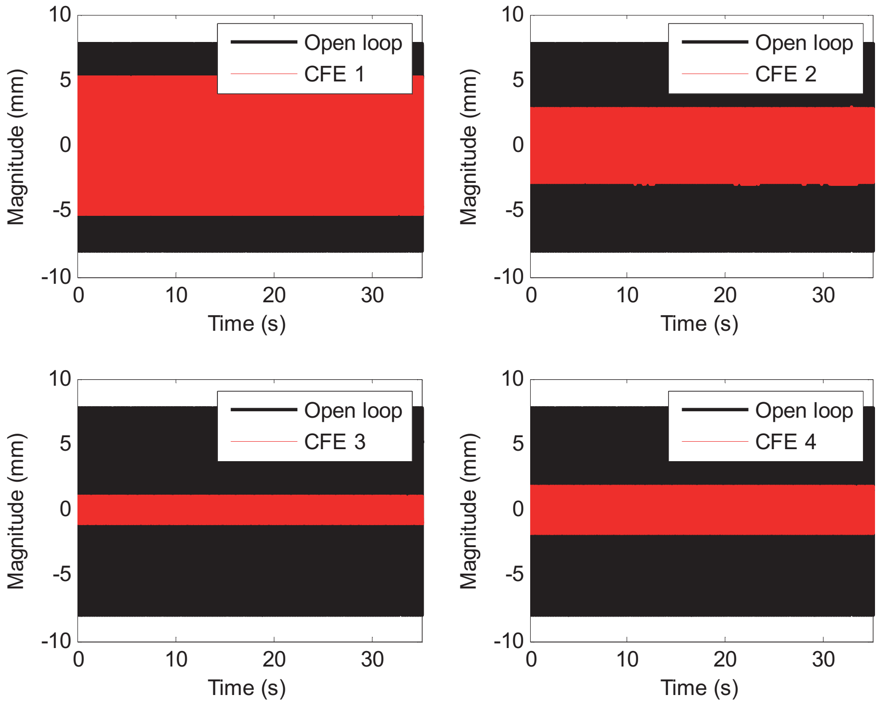


Figure 6. Experimental forced vibration responses of the smart beam at its first resonance frequency.

The experimentally obtained open and closed loop frequency response curves of the smart beam are given in Figure 7. It has been determined that the closed loop resonance frequencies for the controllers with CFE1, CFE2 and CFE3 cases shifted to 6.875 Hz whereas the CFE4 case did not undergo a resonance shift. The attenuation levels of the control cases are defined in Equation 8. These levels are demonstrated in Figure 8 at the shifted frequencies and at the open loop resonance frequency.

$$\text{Attenuation Level} = [\text{Open Loop Vibration Level}] - [\text{Closed Loop Vibration Level}] \quad (8)$$

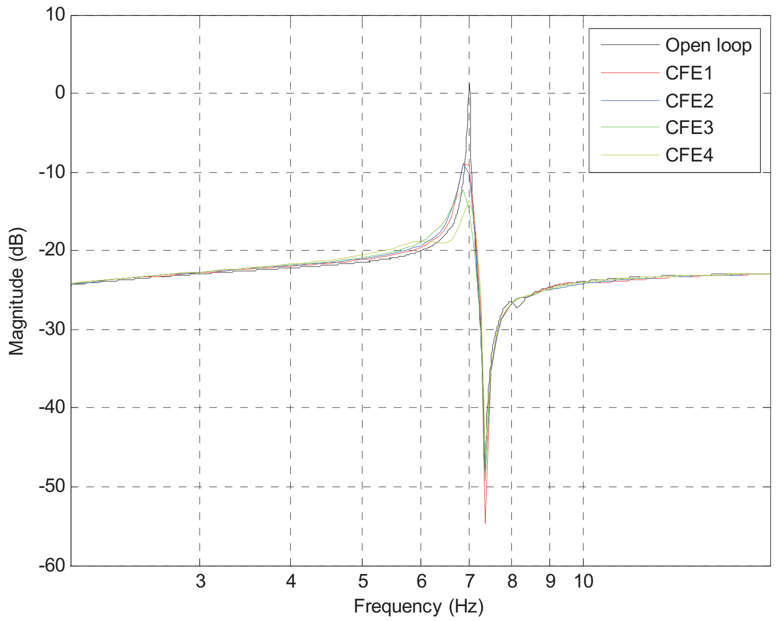


Figure 7. Open and closed loop experimental frequency responses of the smart beam.

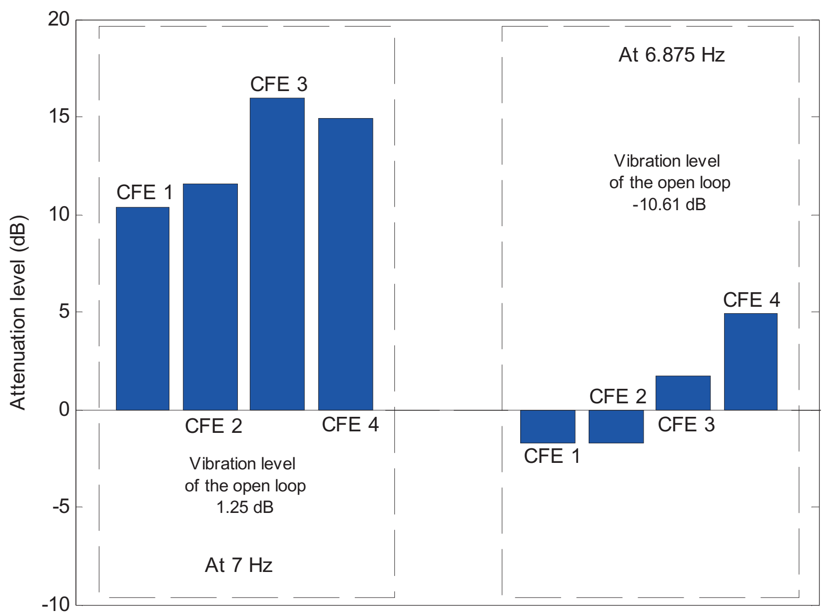
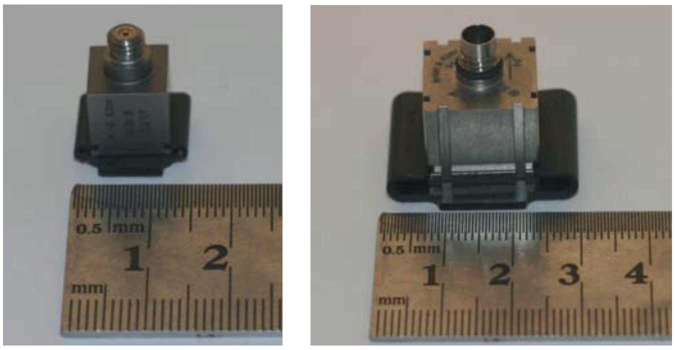


Figure 8. Attenuation levels at shifted resonance frequencies of the smart beam.

6 EXPERIMENTAL ROBUSTNESS TESTS OF THE SMART BEAM

The robustness tests are usually conducted by added masses (Onat 2007, Onat 2009). This is due to the fact that the experiments were easy to conduct and at the same time resonance shifts can easily be done. In this study two separate robustness tests were conducted by adding two different masses (i.e. various accelerometers in different sizes) to the free end of the smart beam for each of the different controllers. Figure 9 shows these accelerometers which were used as concentrated mass during the experimental work.



(a) (b)

Figure 9. Added masses used in the robustness tests (a) Single –axis accelerometer of 5.23 gram (b) Three-axis accelerometer of 17.54 gram.

Initially a 5.23 gram point mass (Fig. 9a) was added. Figure 10 gives the time response for 8 mm tip displacement for four different CFE approaches. It can be seen that the increase in CFE order results in better settlement time.

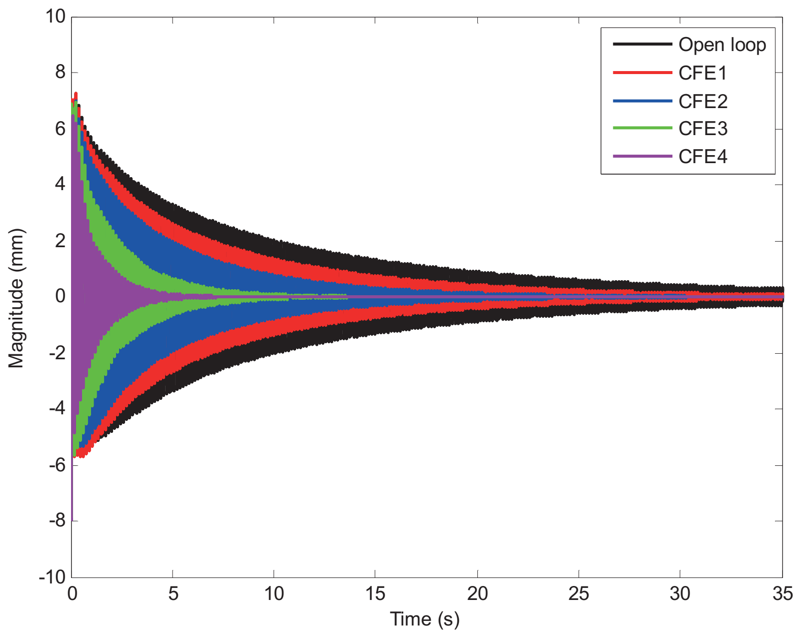


Figure 10. Experimental free vibration responses of the smart beam having 5.23 gram tip mass.

Then, the frequency domain responses are shown in Figure 11 for the same tip mass of Figure 10. Figure 11 indicates that the open loop resonance value reduces to 6.547 Hz and CFE1, CFE2 and CFE3 controllers reduce the resonance values to 6.531, 6.500 and 6.469 Hz values respectively. CFE4 application did not affect the open loop resonance value. The highest vibratory level of the open loop is 4.35 dB. The controllers, with ascending order, suppress that to 0.03 dB, -1.36 dB, -6.10 dB and -14.50 dB values respectively.

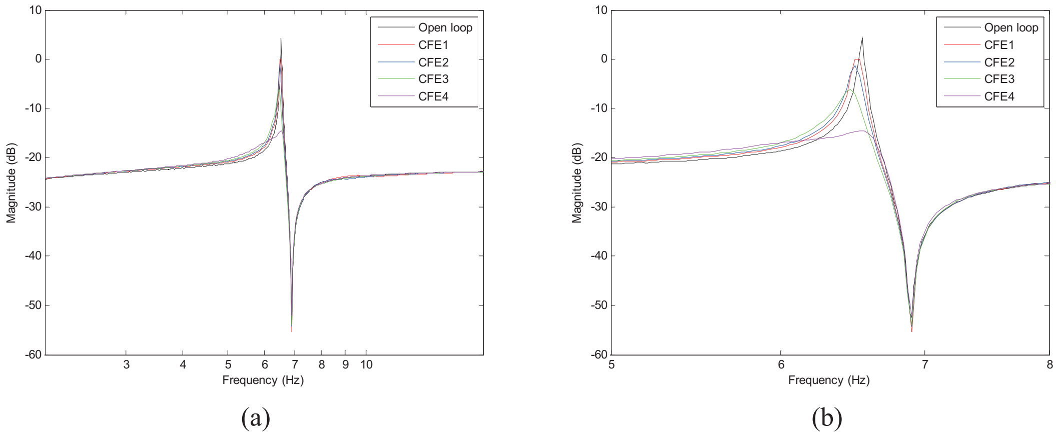


Figure 11. Open and closed loop experimental frequency responses of the smart beam having 5.23 gram tip mass (a) Response between 2 – 18 Hz (b) Zoomed response between 5 – 8 Hz.

The experiments were repeated for a second tip mass of 17.54 gram (Fig. 9b) and the experimentally obtained values are given in Figures 12 and 13 for the time domain and the frequency domain analyses respectively.

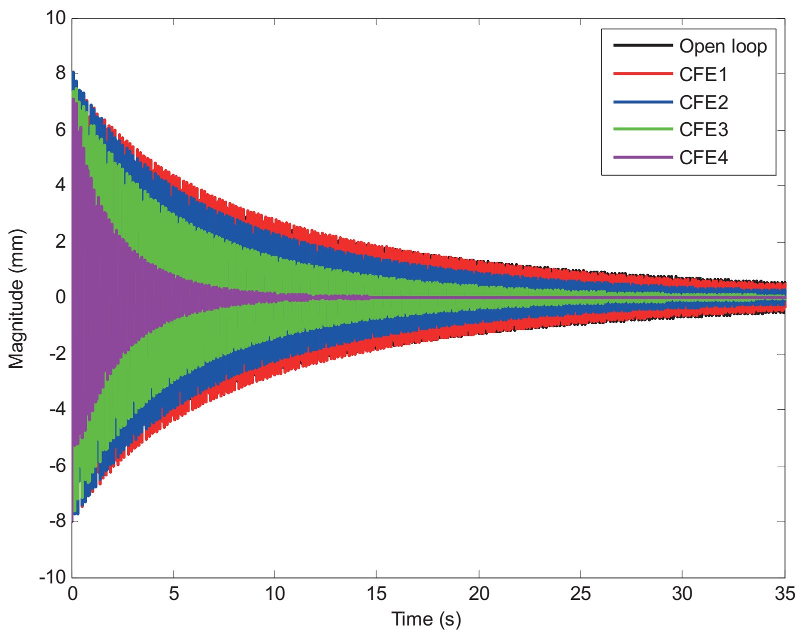
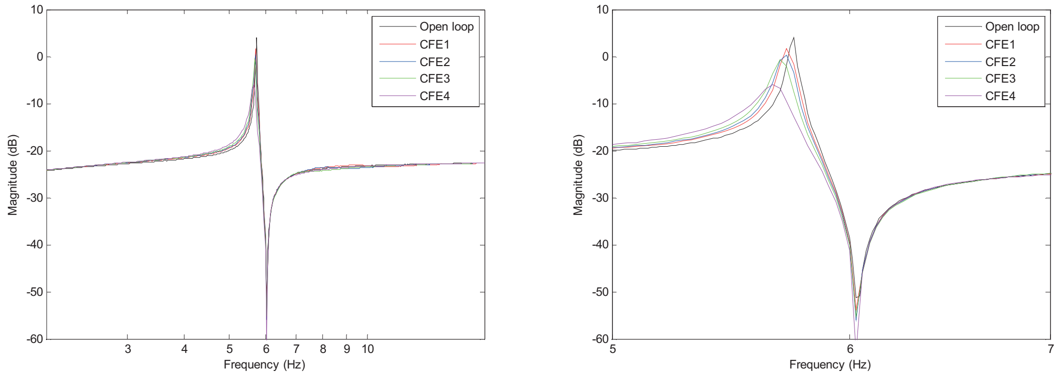


Figure 12. Experimental free vibration responses of the smart beam having 17.54 gram tip mass.



(a) (b)

Figure 13. Open and closed loop experimental frequency responses of the smart beam having 17.54 gram tip mass (a) Response between 2 – 18 Hz (b) Zoomed response between 5 – 7 Hz.

Figure 13 gives a 5.750 Hz open loop resonance value. CFE1 and CFE2 controllers shift the closed loop responses to 5.719 Hz. CFE3 and CFE4 controllers, on the other hand, shift to 5.688 Hz and 5.656 Hz respectively. The highest vibratory level of the open loop is 4.044 dB. The controllers, with ascending order, suppress that to 1.620 dB, 0.361 dB, -0.775 dB and -6.038 dB values respectively. Comparison of Figures 11 and 13 indicates that the increase in the added mass, as expected, reduces the resonance frequencies of respective cases.

7 CONCLUSION

Implementation of a fractional controller which was considered by using various order approaches of the CFE method was presented for suppressing the first flexural resonance level of a smart beam. Realizations of the designed fractional controllers were conducted by using the first, second, third and fourth order approaches of the CFE method. The experimental results which were obtained in both time and frequency domain shows that increase in the order of the controllers also increased the performance of the controller.

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