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AN EXPERIMENTAL PERFORMANCE EVALUATION FOR THE SUPPRESSION OF VIBRATIONS OF THE SECOND MODE OF A SMART BEAM

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ABSTRACT

In this study the fractional controllers, which were realized by the different degrees of the Continued Fractional Expansion (CFE) method, were experimentally evaluated for the suppression of the vibrations of the second mode of a smart beam. The smart beam is equipped with PZT patches and these patches were used both as actuator and/or sensor. The control strategy was based on the fractional derivation of the measurement signal which was the displacement values and filtering that signal by using a filter which was designed to characterise the dynamic properties of the second mode of the smart beam. The experimental results showed that when the controller was realized with a higher fractional derivation degree, better vibration suppression was provided for the second mode.

INTRODUCTION

This study gives the analysis, design and applications of the fractional order differentiators for the active vibration control of the second mode of a smart beam. The fractional order differentiators are the examples of fractional order systems. The fractional order systems are described by the fractional order differential equations [1, 2]. Fractional order differentiators are used to compute the fractional order time derivative of the given signal [3-6]. Geometrical and physical interpretations of fractional order differentiators are widely discussed in literature [7-11]. Fractional order control systems have transfer functions with fractional derivatives s^{α} and fractional integrals $s^{-\alpha}$ where α€R. It is not very easy to compute the frequency and time domain behaviours of such fractional order transfer functions with available software packages. It is well known that the simulation programs have been prepared to deal with integer power only. Although, there are some recent works dealing with implementation of a controller using fractance device [12], this area also needs further studies since an electronic component to implement fractional order systems is not, recently, commercially available. Therefore, the problem of integer order approximations of fractional order functions becomes a very important one to be attempted. A fractional transfer function can be replaced with an integer order transfer function which has almost the same behaviours with the actual transfer function. There are various methods [13-20] for computing the integer order approximations of the fractional order operators such as s^{α} or $s^{-\alpha}$. One of the most widely encountered approximations for fractional order systems is the CFE method [21].

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A fractional order controller which was designed by using a fourth degree approach of CFE method was successfully applied for the active control of the first flexural mode vibrations of a smart beam [24]. A variety of different degree approaches of CFE method was also applied, again in order to suppress the first flexural mode vibrations of the same smart beam. The performance and the robustness characteristics were evaluated. It was shown that the increase in the approach value of CFE method significantly increased the performance of the developed controller [25].

In this study a fractional order controller, developed by using the CFE method, was designed and implemented for the suppression of the second flexural mode vibration of a smart beam. The first, second, third and fourth degree approaches of CFE method were studied together with an integer counterpart for the performance analysis of the controller. Experimentally obtained results were presented for the suppression of the free and forced vibrations

SMART BEAM

The smart beam studied was a cantilever aluminium beam with eight surface bonded Lead-Zirconate-Titanate (PZT) patches and is shown in Figure 1. A thin isolation layer was placed between the aluminium beam and PZT patches hence each PZT patch might independently be employed as a sensor and/or an actuator [22]. Figure 2 and Figure 3 show the experimental setup used in the study and the frequency response of the smart beam covering the first two flexural resonance frequencies respectively.

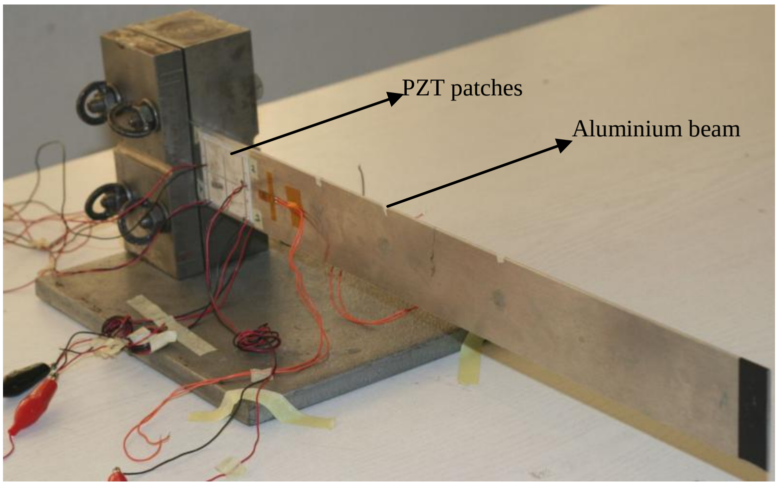


Figure 1: *The smart beam used in the study*

The smart beam was then harmonically excited in a frequency range to cover the second resonance frequency (approx. at 41.25 Hz) with piezoelectric patches acting as actuators and the response of the smart beam was obtained from a different single piezoelectric sensor patch acting as a sensor in order to obtain the necessary experimental frequency response of the smart beam for the system identification.

The mathematical model of the smart beam was obtained by processing the measured frequency response data. By using MATLAB's "fitsys" command located in μ Analysis and Synthesis Toolbox the transfer function of the smart beam was determined [24]. MATLAB "fitsys" command builds a state-space model based on estimated transfer function. Transfer function of the smart beam is estimated within the frequency range between 30 Hz and 50 Hz. This frequency range included the second flexural mode (approx. at 41.25 Hz) of the smart beam.

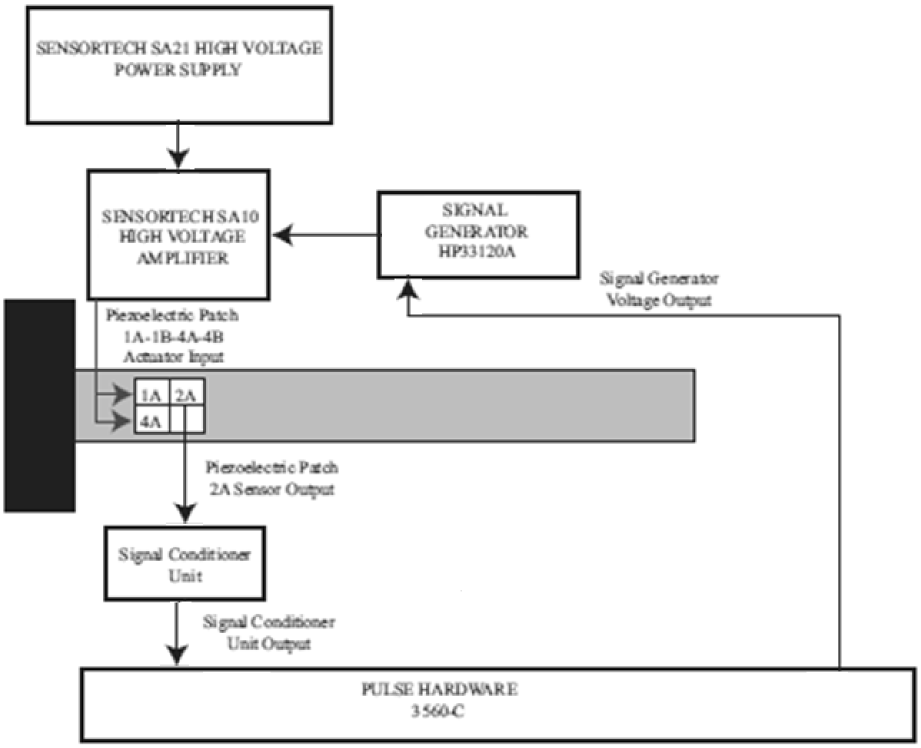


Figure 2: *The experimental setup used in the study*

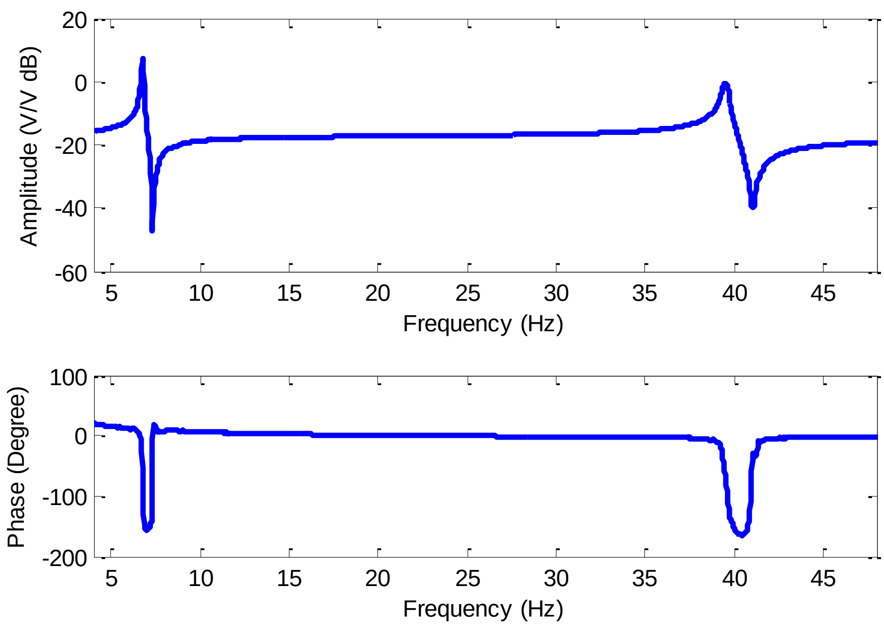


Figure 3: *Frequency response of the smart beam covering the first two flexural modes*

The determined 6^{th} order transfer function of the smart beam is given in Equation 1 and Figure 4 shows the experimentally measured and analytically estimated transfer functions of the smart beam.

$$G(s)=\frac{0.05947\cdot s^6+0.701\cdot s^5+1.225\cdot 10^4\cdot s^4+9.642\cdot 10^4\cdot s^3+8.405\cdot 10^8\cdot s^2+3.314\cdot 10^9\cdot s+1.922\cdot 10^{13}}{s^6+12.74\cdot s^5+2.028\cdot 10^5\cdot s^4+1.716\cdot 10^6\cdot s^3+1.371\cdot 10^{10}\cdot s^2+5.779\cdot 10^{10}\cdot s+3.089\cdot 10^{14}} \quad (1)$$

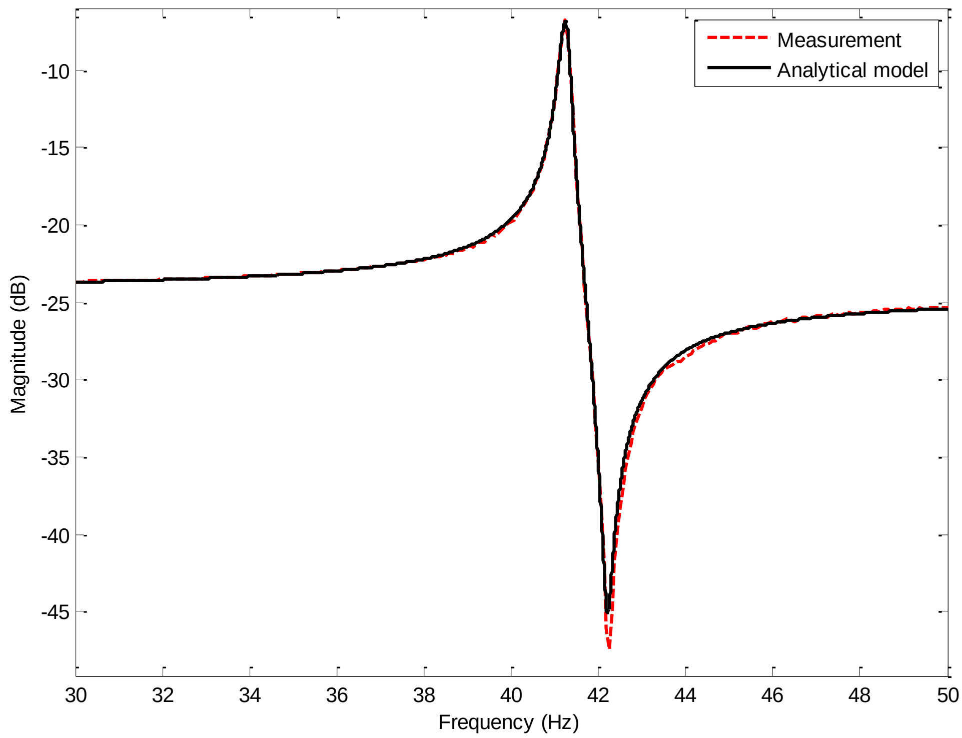


Figure 4: *Experimentally measured and analytically estimated transfer functions of the smart beam around its second mode*

CONTROL DESIGN METHOD

In this study, the differential effect was included as the fractional one and the controller for active vibration suppression was synthesized in two steps. First, the fractional differential effect of the smart beam was derived from the measurement signal by using the fractional derivative effect s^μ. In this study different approximations for s^μ was considered by using first, second, third and fourth degree approach of CFE method. These approximations are given in Equations 2 to 5 in ascending order [25, 26].

$$s^\mu \cong \frac{(1+\mu)\cdot s+(1-\mu)}{(1-\mu)\cdot s+(1+\mu)} \quad (2)$$

$$s^\mu \cong \frac{(\mu^2+3\cdot\mu+2)\cdot s^2+(-2\cdot\mu^2+8)\cdot s+(\mu^2-3\cdot\mu+2)}{(\mu^2-3\cdot\mu+2)\cdot s^2+(-2\cdot\mu^2+8)\cdot s+(\mu^2+3\cdot\mu+2)} \quad (3)$$

$$s^{\mu} \cong \frac{\left(\mu^3+6\cdot\mu^2+11\cdot\mu+6\right)\cdot s^3+\left(-3\cdot\mu^3-6\cdot\mu^2+27\cdot\mu+54\right)\cdot s^2 +\left(3\cdot\mu^3-6\cdot\mu^2-27\cdot\mu+54\right)\cdot s+\left(-\mu^3+6\cdot\mu^2-11\cdot\mu+6\right)}{\left(-\mu^3+6\cdot\mu^2-11\cdot\mu+6\right)\cdot s^3+\left(3\cdot\mu^3-6\cdot\mu^2-27\cdot\mu+54\right)\cdot s^2 +\left(-3\cdot\mu^3-6\cdot\mu^2+27\cdot\mu+54\right)\cdot s+\left(\mu^3+6\cdot\mu^2+11\cdot\mu+6\right)} \quad (4)$$

$$s^{\mu} \cong \frac{\left(\mu^4+10\cdot\mu^3+35\cdot\mu^2+50\cdot\mu+24\right)\cdot s^4+\left(-4\cdot\mu^4-20\cdot\mu^3+40\cdot\mu^2+320\cdot\mu+384\right)\cdot s^3 +\left(6\cdot\mu^4-150\cdot\mu^2+864\right)\cdot s^2+\left(-4\cdot\mu^4+20\cdot\mu^3+40\cdot\mu^2-320\cdot\mu+384\right)\cdot s +\left(\mu^4-10\cdot\mu^3+35\cdot\mu^2-50\cdot\mu+24\right)}{\left(\mu^4-10\cdot\mu^3+35\cdot\mu^2-50\cdot\mu+24\right)\cdot s^4+\left(-4\cdot\mu^4+20\cdot\mu^3+40\cdot\mu^2-320\cdot\mu+384\right)\cdot s^3 +\left(6\cdot\mu^4-150\cdot\mu^2+864\right)\cdot s^2+\left(-4\cdot\mu^4-20\cdot\mu^3+40\cdot\mu^2+320\cdot\mu+384\right)\cdot s +\left(\mu^4+10\cdot\mu^3+35\cdot\mu^2+50\cdot\mu+24\right)} \quad (5)$$

Then a filter was designed in order to represent the dynamic characteristics of the second flexural mode of the smart beam. The designed filter, H(s), is given in Equation 6.

$$H(s)=\frac{1}{s^2+27.65\cdot s+76430} \quad (6)$$

The block diagram of the closed loop system is given Figure 5. X(s), Y(s) and K stand for the system input, the system output and the controller gain in Laplace domain in order.

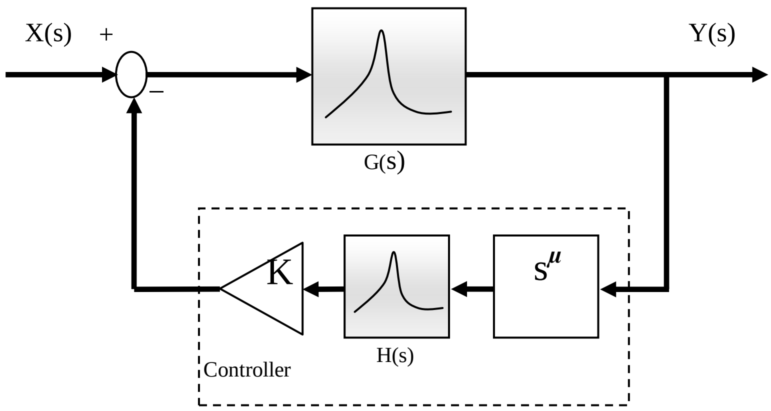


Figure 5: *Control block diagram*

SIMULATIONS

Figure 6 gives the closed loop simulation results of the smart beam at the immediate vicinity of the second flexural mode for different differential effect, μ, values. At the selected range although μ=0.1 to μ=0.8 results are comparable, the value after μ=0.9 shows better suppression. Hence μ=0.99 was selected in further analysis.

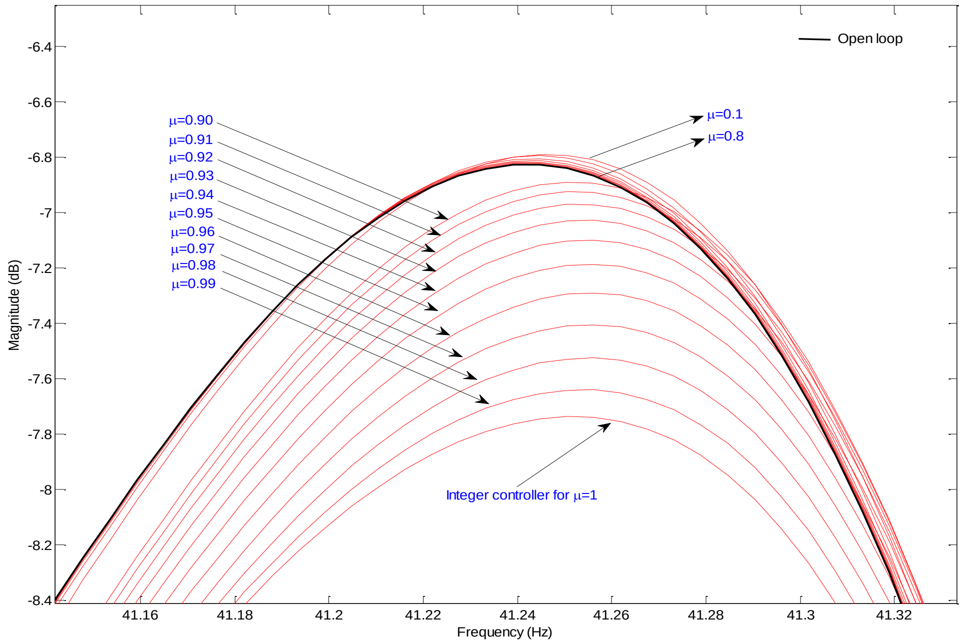
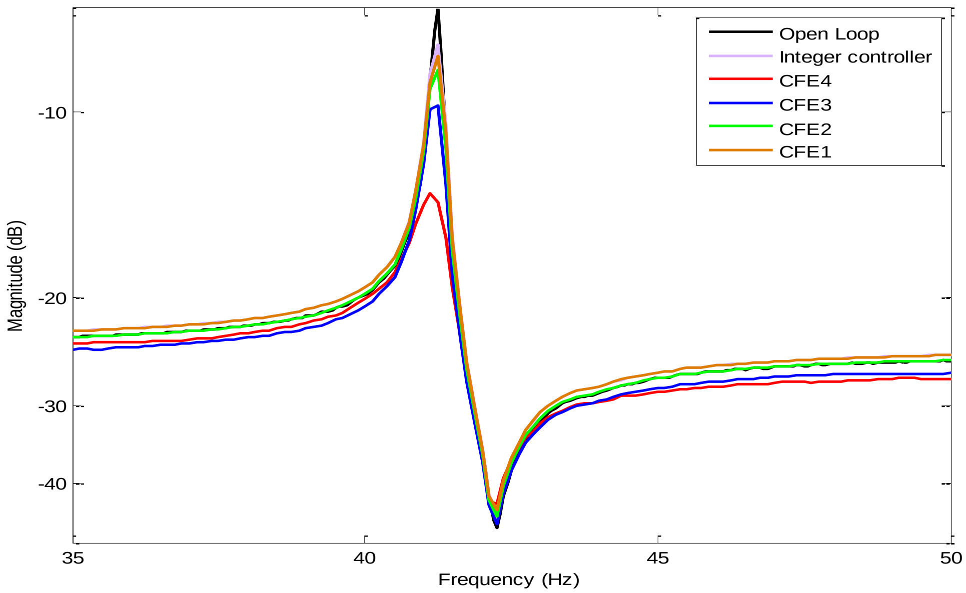


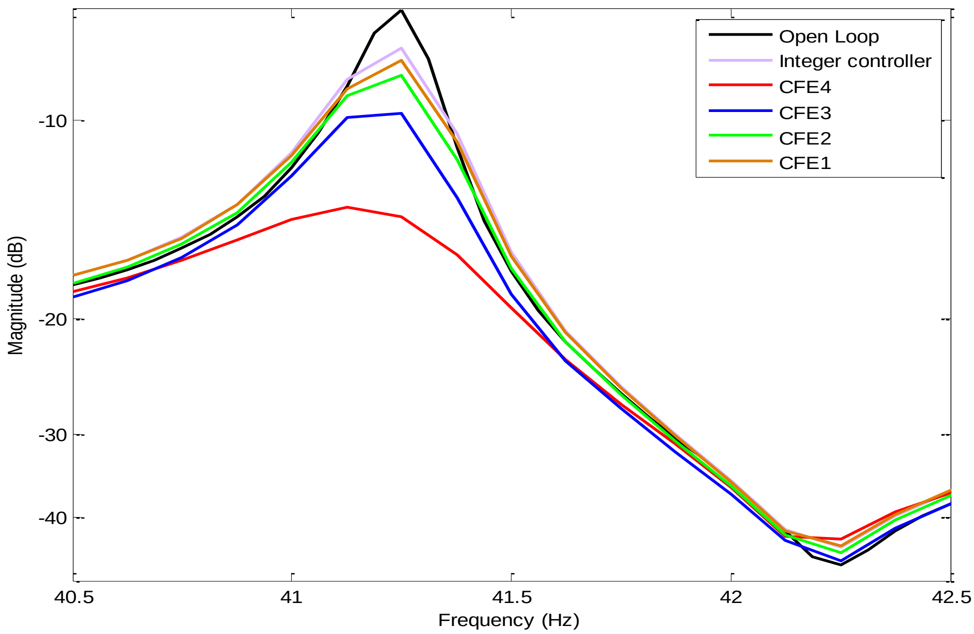
Figure 6: *Closed loop Simulation results of the smart beam around its second mode*

EXPERIMENTAL RESULTS

The fractional controller corresponding to μ=0.99 was realized by using the 1st, 2nd, 3rd and 4th degree approximations of the CFE method. The controllers were applied to the smart beam and the experimentally determined closed loop frequency responses are given in Figure 7. The figure also represents the results of the open loop response and of the integer order controller corresponding to μ=1. Figure 7 clearly shows that a fractional controller realized with a higher order approximation provides better suppression as compared to the integer order controller.



(a)



(b)

Figure 7: *a) Experimental frequency responses of the smart beam around its second mode b) zoomed frequency responses*

The attenuation levels of the control cases are then calculated by using Equation 7 and these levels are also demonstrated in Figure 8 both at the open and closed loop resonance frequencies.

$$\text{Attenuation Level} = [\text{Open Loop Vibration Level}] - [\text{Closed Loop Vibration Level}] \quad (7)$$

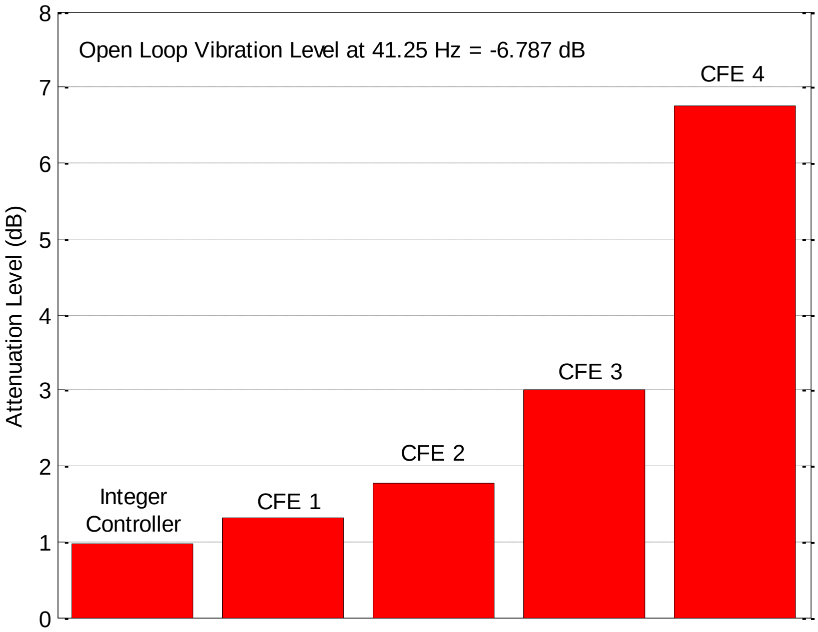


Figure 8: *Attenuation levels of the smart beam.*

Time domain response of the smart beam was obtained by exciting with a sinusoidal disturbance whose frequency is equal to second resonance frequency of 41.25 Hz. Figure 8 represents the time domain responses of the smart beam obtained at the second resonance frequency. It can be seen that the fractional controller realized by the 4th degree approach of CFE can achieve an approximately 45% reduction in the resonant vibration level.

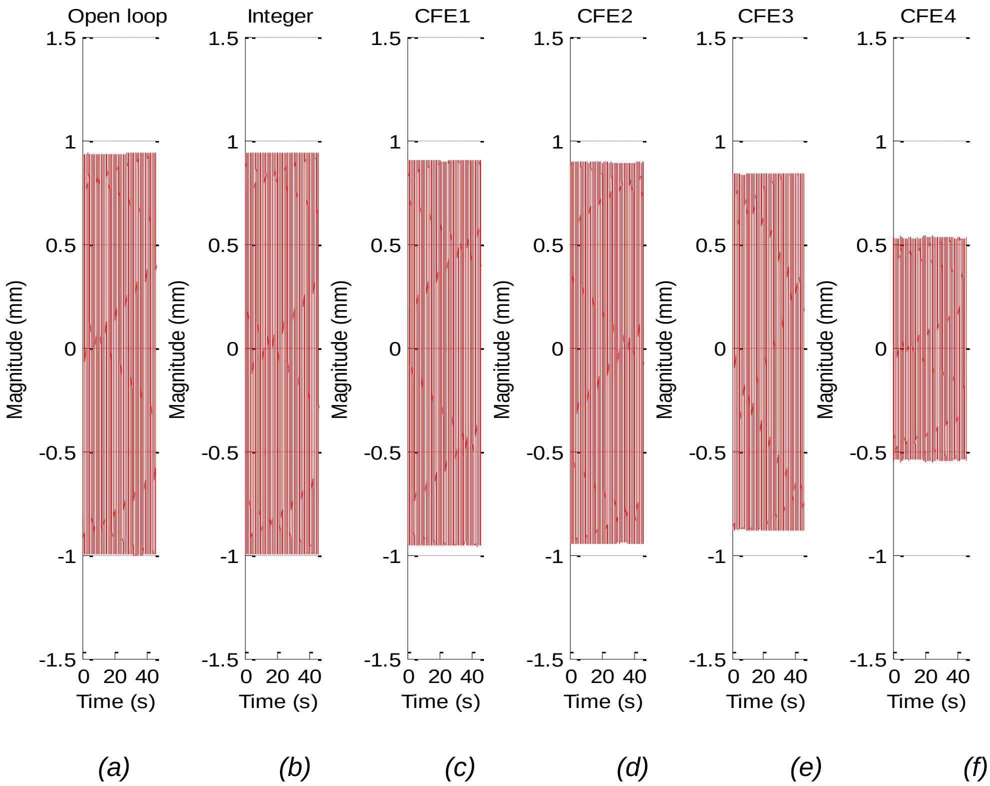


Figure 8: *Experimental time domain responses* of the smart beam at its second mode *(a)Open loop (b) Integer (c) CFE 1st degree (d) CFE 2nd degree (e) CFE 3rd degree (f) CFE 4th degree*

CONCLUSION

The design and implementation of a fractional controller was presented. The controller was considered by using different degree approaches of the CFE method and was intended to suppress the second flexural resonance level of a smart beam. During the controller design the fractional value of the differentiator was used as a design variable. It was shown that the increase in the approach degree of the CFE method provided an improvement in the suppression of the vibrational response of the second mode of the smart beam.

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