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DESIGN OF AN LPV BASED FRACTIONAL CONTROLLER FOR THE VIBRATION SUPPRESSION OF A SMART BEAM

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ABSTRACT

One of the major problems encountered in the active vibration control of aircraft wings is the changing mass due to the in-flight fuel consumption. In this study, a Linear Parameter Varying (LPV) based fractional controller is designed for the suppression of the flexural vibrations of a smart beam. The designed controller is sensitive to the varying mass properties. The smart beam studied was a cantilever aluminium beam with eight surface bonded Lead-Zirconate-Titanate (PZT) patches. The smart beam was excited at its first resonance frequency (approx. at 7 Hz) with a group of piezoelectric actuator patches and the response of the smart beam was monitored from a single piezoelectric sensor patch in order to obtain the necessary experimental frequency response for the system identification. The control strategy was based on the fractional derivation of the measurement signal and filtering that signal by using a developed filter which was designed to characterize the dynamic properties of the first mode of the smart beam. The filter is designed as an LPV filter and is scheduled by the mass of the smart beam. The applications were conducted by attaching different masses to the tip of the smart beam. Both time domain and frequency domain responses were analysed. It was shown that the designed controller was satisfactorily capable of suppressing the smart beam vibrations even at the presence of varying mass characteristics.

Keywords: Smart beam, Vibration control, LPV model.

INTRODUCTION

Gain scheduling is a widely used technique for controlling certain classes of linear or nonlinear time-varying systems. Rather than seeking a single robust linear time invariant (LTI) controller for the entire operating range, gain scheduling consists in designing an LTI controller for each operating point and in switching controller when the operating conditions change [1,2]. An aircraft wing with changing internal fuel during the flight could be considered as a linear parameter varying (LPV) system as the change in fuel over time results changing in dynamic properties of the wing such as resonance frequencies.

Therefore in this study, since the wings of an aircraft can be considered as cantilever structures, the suppression of the first flexural resonance vibrations of a smart cantilever aluminium beam-like structure with eight surface bonded PZT patches was achieved under various loading conditions via designed linear parameter varying based fractional controller. The active vibration control was basically performed by using an LPV filter. That filter was scheduled by changing mass of the internal fuel. The order of the fractional differentiator was also chosen as a design parameter in the current study.

SMART BEAM MODEL

The smart beam given in Figure 1 is a cantilever aluminium beam having the dimensions of 490 x 51 x 2 mm and eight surface bonded SensorTech - BM500 (25 x 20 x 0.5 mm) PZT (Lead - Zirconate -Titanate) patches (Sensor, 2002). A thin isolation layer is placed between the aluminium beam and PZT patches, so that each PZT patch may be employed as a sensor or an actuator independently.

Fig. 1: Smart beam used in the study

The mathematical models of the smart beam with and without tip mass loading (i.e. loaded and unloaded cases respectively) were obtained by processing the measured frequency response data. By using MATLAB’s “fitsys” command located in “μ Analysis and Synthesis Toolbox” the transfer function of the smart beam was determined. MATLAB “fitsys” command builds a state-space model based on estimated transfer function. The transfer functions of the smart beam for both unloaded and loaded (with mass) cases were estimated within the frequency range between

2 Hz and 18 Hz which includes the first flexural resonance frequency of the smart beam. The equations (1) and (2) present the transfer functions of the smart beam for unloaded and loaded (tip mass=17.54 [gr]) cases respectively.

$$G(s) = \frac{0.06449 \cdot s^6 + 13.42 \cdot s^5 + 288.7 \cdot s^4 + 5.466 \cdot 10^4 \cdot s^3 + 3.548 \cdot 10^5 \cdot s^2 + 5.55 \cdot 10^7 \cdot s + 7.102 \cdot 10^7}{s^6 + 191.6 \cdot s^5 + 6085 \cdot s^4 + 7.418 \cdot 10^5 \cdot s^3 + 1.211 \cdot 10^7 \cdot s^2 + 7.179 \cdot 10^8 \cdot s + 7.89 \cdot 10^9} \quad (1)$$

$$G_M(s) = \frac{0.07122 \cdot s^6 + 3.638 \cdot s^5 + 207.7 \cdot s^4 + 9938 \cdot s^3 + 1.672 \cdot 10^5 \cdot s^2 + 6.773 \cdot 10^6 \cdot s + 2.269 \cdot 10^7}{s^6 + 56.37 \cdot s^5 + 3292 \cdot s^4 + 1.468 \cdot 10^5 \cdot s^3 + 3.472 \cdot 10^6 \cdot s^2 + 9.556 \cdot 10^7 \cdot s + 1.147 \cdot 10^9} \quad (2)$$

Here, G(s) and $G_M(s)$ correspond to the transfer functions for unloaded and loaded cases respectively. Equation (3), on the other hand, shows the state-space representation of the smart beam LPV model comprising the loading mass ‘m’ as a variable.

$$\begin{aligned} \dot{x} &= A(m) \cdot x + B(m) \cdot (u + w) \\ y &= C(m) \cdot x + D(m) \cdot (u + w) \end{aligned} \quad (3)$$

In this equation, x, y, u, and w are the state, output, control input and disturbance vectors and A, B, C and D are the state space model matrices. This model was obtained by using the corner transfer functions given in equations (1) and (2) and the “Linear Matrix Inequalities (LMI) Control Toolbox” of MATLAB software. The smart beam model for any intermediate loading condition between the unloaded and the fully loaded cases, can also be obtained at each required instant of the vibratory motion by scheduling this particular LPV model.

An excitation was then given as a swept sine signal from 2 Hz to 18 Hz with 5 V peak-to-peak value generated by HP33120A signal generator. It can be observed from the sine sweep tests performed on smart beam that the fundamental resonance frequencies for unloaded and loaded (tip mass=17.54 [gr]) cases are found approximately as 7 Hz and 5.75 Hz respectively. Figure 2 and Figure 3 present frequency domain results of the experimentally measured and analytically estimated transfer functions of G(s) and $G_M(s)$ of the smart beam around its first mode. It can also be observed from the figures that the 6^{th} order transfer functions adequately represent the dynamic behaviour of the smart beam. Following these analytical and experimental works, the frequency response of the LPV model of the smart beam was also obtained and given in Figure 4 for different tip loading conditions. The shift in frequencies due to addition of various mass is tabulated in Table 1. The values in parenthesis indicate the percentage reduction in frequency from the unloaded case.

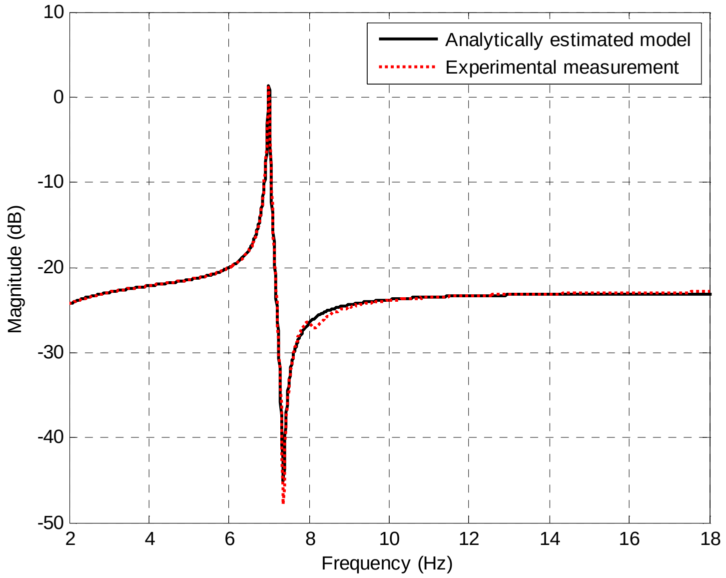

Fig. 2: Analytically estimated G(s) and experimentally measured transfer functions of the LPV modelled smart beam around its first mode

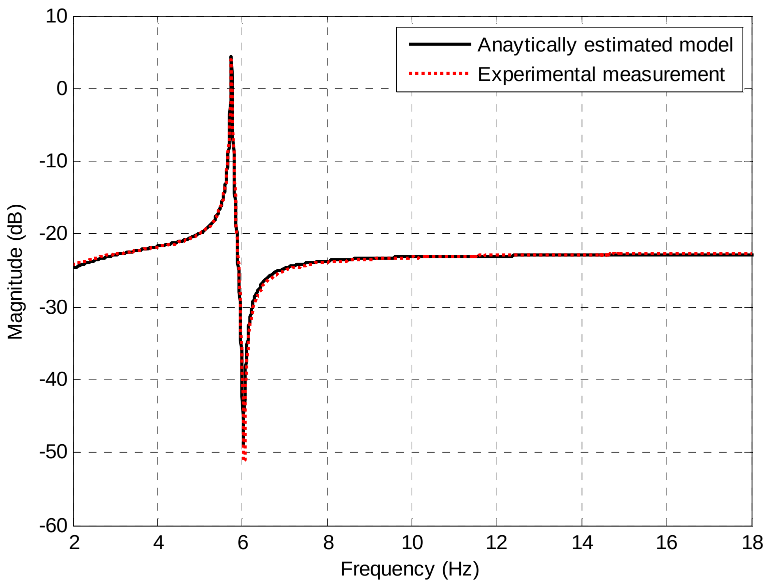

Fig. 3: Analytically estimated $G_M(s)$ and experimentally measured transfer functions of the LPV modelled smart beam around its first mode (tip mass=17.54 [gr])

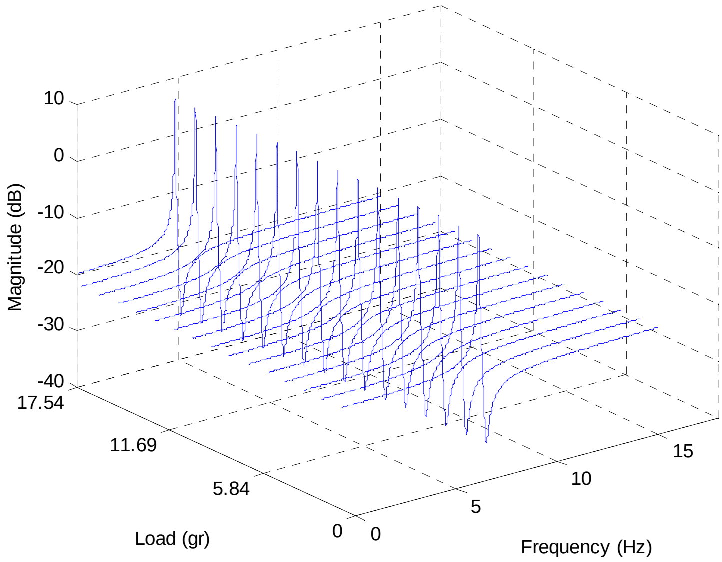

Fig. 4: Frequency responses of LPV modelled smart beam for various tip loads

Table 1: Resonance frequencies of the LPV modelled smart beam under various tip loads

Mass [gr]	Frequency [Hz]	Mass [gr]	Frequency [Hz]
Unloaded	7.00	9.35	6.35 (9.29)
1.16	6.92 (1.14)	10.52	6.27 (10.43)
2.33	6.84 (2.28)	11.69	6.19 (11.57)
3.50	6.76 (3.43)	12.86	6.10 (12.86)
4.67	6.68 (4.57)	14.03	6.02 (14.00)
5.84	6.60 (5.71)	15.20	5.92 (15.43)
7.01	6.52 (6.86)	16.37	5.84 (16.57)
8.18	6.44 (8.00)	17.54	5.75 (17.86)

CONTROL STRATEGY

An active vibration controller development for a system is similar to the determination of a suitable damping ratio for the same system. That stems from the fact that the differential effect is analogous with the velocity and hence the knowledge of the differential effect becomes important for the controller design. In this study the fractional differential effect was considered.

The control strategy is based on the fractional derivation of the measurement signal and filtering that signal by using a developed filter which is designed to characterise the dynamic properties of the first mode of the smart beam. The filter is designed as an LPV filter and is scheduled by the mass of the smart beam. Block diagram of the designed controller is shown in Figure 5.

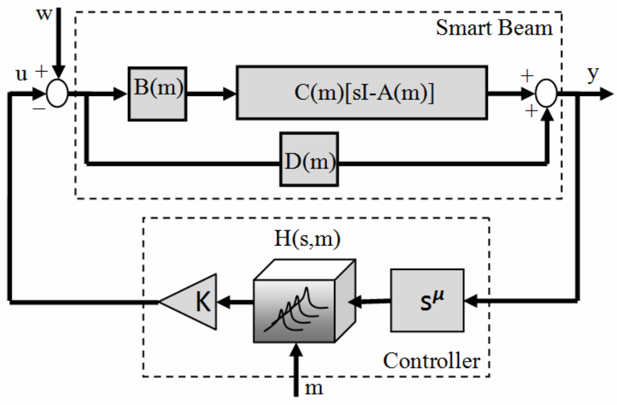

Fig. 5: LPV based fractional closed loop system

In Figure 5, H(s,m) is the LPV filter which characterises the first mode of the smart beam with the addition of the tip mass. This function is obtained by using MATLAB LMI Control Toolbox and the resultant corner transfer functions are given in equations (4) and (5) for H(s) and $H_M(s)$ which respectively characterise the unloaded and fully loaded cases in the first mode of the smart beam. The linear interpolation was used for the intermediate loading cases in order to find the corresponding H(s,m).

$$H(s) = \frac{1}{s^2 + 5.1 \cdot s + 2590.2} \tag{4}$$

$$H_M(s) = \frac{1}{s^2 + 4.3 \cdot s + 1825.5} \tag{5}$$

The fractional differential effect (i.e. fractional velocity signal) of the smart beam was derived from the measurement signal by using the fractional derivative effect s^μ. The s^μ was considered by using a fourth degree approach of Continued Fractional Expansion method and is given in Equation 6 [3-6].

$$s^\mu \cong \frac{\begin{array}{l}(\mu^4 + 10\cdot\mu^3 + 35\cdot\mu^2 + 50\cdot\mu + 24)\cdot s^4 + (-4\cdot\mu^4 - 20\cdot\mu^3 + 40\cdot\mu^2 + 320\cdot\mu + 384)\cdot s^3 \\ + (6\cdot\mu^4 - 150\cdot\mu^2 + 864)\cdot s^2 + (-4\cdot\mu^4 + 20\cdot\mu^3 + 40\cdot\mu^2 - 320\cdot\mu + 384)\cdot s \\ + (\mu^4 - 10\cdot\mu^3 + 35\cdot\mu^2 - 50\cdot\mu + 24)\end{array}}{\begin{array}{l}(\mu^4 - 10\cdot\mu^3 + 35\cdot\mu^2 - 50\cdot\mu + 24)\cdot s^4 + (-4\cdot\mu^4 + 20\cdot\mu^3 + 40\cdot\mu^2 - 320\cdot\mu + 384)\cdot s^3 \\ + (6\cdot\mu^4 - 150\cdot\mu^2 + 864)\cdot s^2 + (-4\cdot\mu^4 - 20\cdot\mu^3 + 40\cdot\mu^2 + 320\cdot\mu + 384)\cdot s \\ + (\mu^4 + 10\cdot\mu^3 + 35\cdot\mu^2 + 50\cdot\mu + 24)\end{array}} \tag{6}$$

EXPERIMENTAL RESULTS

The smart beam is given an initial 8 mm tip deflection and the ensuing motion is measured for open and closed loop time responses. Figure 6 and Figure 7 show those responses for unloaded and fully loaded (tip mass=17.54 [gr]) cases respectively. For both cases, the controller successfully suppresses the vibrations of the first flexural mode within approximately 6 seconds.

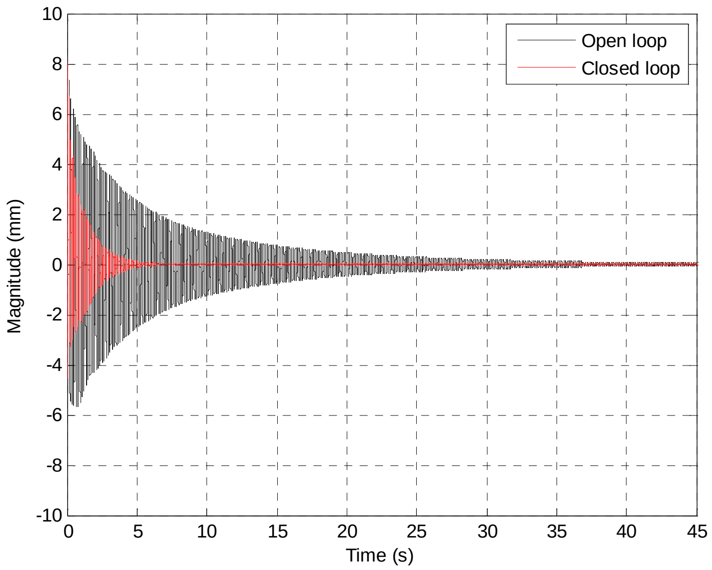

Fig. 6: Time domain response of free vibrations of the LPV modelled smart beam for unloaded case

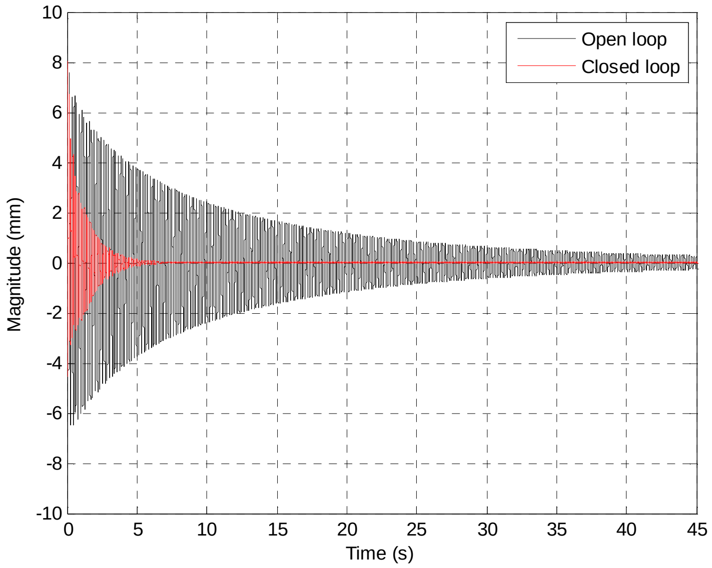

Fig. 7: Time domain response of free vibrations of the LPV modelled smart beam for fully loaded case (tip mass=17.54 [gr])

Analysis of the suppression of the forced vibrations of the smart beam was also performed. The beam was excited at its first resonance frequency both for the unloaded (approximately at 7 Hz) and the fully loaded (approximately at 5.75 Hz) cases by using the PZT patches as actuators. In these experiments, the controller was switched on at the 10^{th} second where the beam was resonating at its first mode and kept active approximately 20 more seconds. Figure 8 and Figure 9 show forced vibrational response of the smart beam in time domain for unloaded and fully loaded cases respectively. The frequency domain results were also obtained and are given in Figure 10 and Figure 11 for unloaded and fully loaded cases respectively.

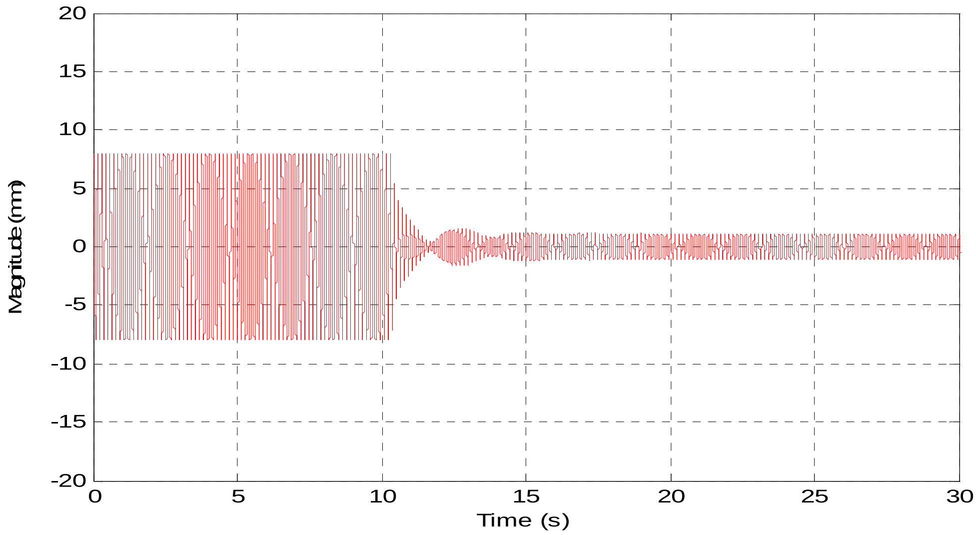

Fig. 8: Time domain response of forced vibrations of the LPV modelled smart beam for unloaded case

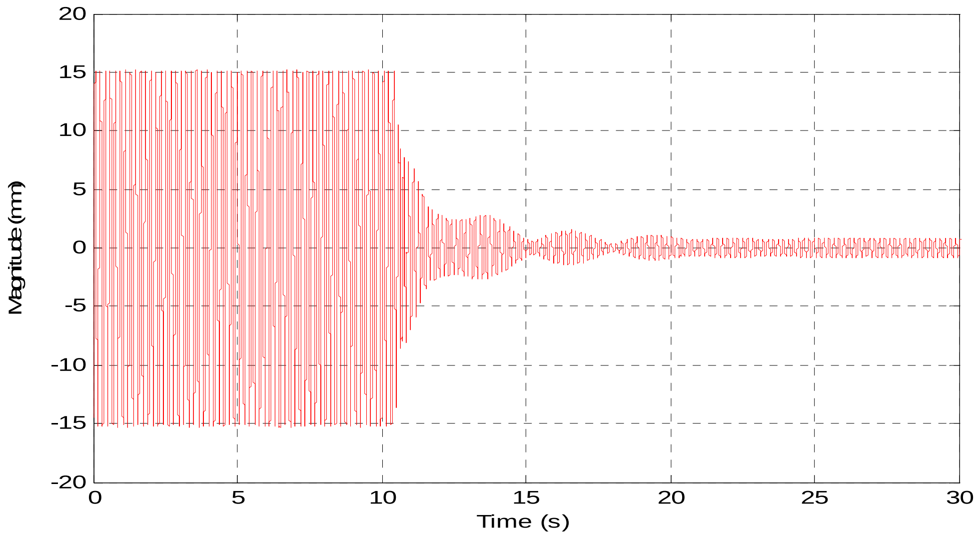

Fig. 9: Time domain response of free vibrations of the LPV modelled smart beam for fully loaded case (tip mass=17.54 [gr])

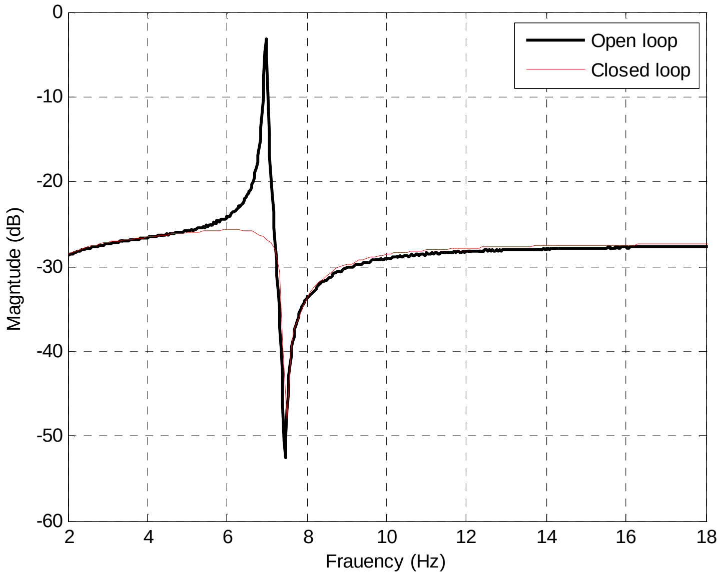

Fig. 10: Open and closed loop frequency responses of the LPV modelled smart beam for unloaded case

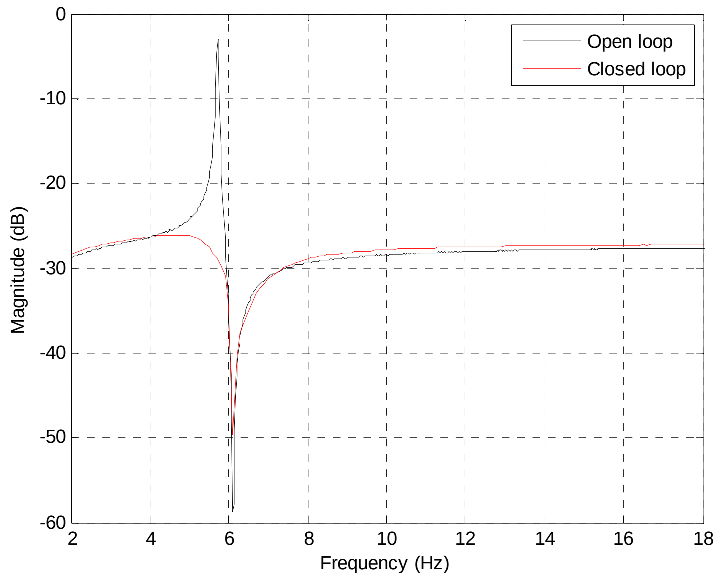

Fig. 11: Open and closed loop frequency responses of the LPV modelled smart beam for fully loaded case (tip mass=17.54 [gr])

It can be observed from Figures 8 to 11 that the controller shows an excellent performance at the first resonance frequencies even without any compromise in the fully loaded case.

CONCLUSION

In this study, the design and implementation of a linear parameter varying (LPV) controller was considered in order to suppress the first flexural vibrations of a smart beam under various tip mass loading. The experimental results, which were obtained both in time and frequency domain, show that the designed controller was capable of reducing the vibration levels successfully for changing mass values.

REFERENCES

1. Onat, C., Kucukdemiral, I. B., Sivrioglu, S., Yuksek, I. "LPV model based gain-scheduling controller for a full vehicle active suspension system", Journal of Vibration and Control, v:13, N:11, 2007, 1629-1666.
2. Onat, C., Kucukdemiral, I. B., Sivrioglu, S., Yuksek, I., and Cansever, G., "LPV gain-scheduling controller design for a nonlinear quarter-vehicle active suspension system", Transactions of the Institute of Measurement and Control, vol.31, no.1, 2009, 71-95.
3. Onat, C., Sahin, M., Yaman, Y.. "Active vibration suppression of a smart beam by using a Fractional Control". 2nd International Conference of Engineering Against Fracture (ICEAF II) 22-24 June, Mykonos, Greece, 2011.
4. Onat, C., Sahin, M., Yaman, Y. "Active vibration suppression of a smart beam via $PI^{\lambda}D^{\mu}$ control". International Workshop on Piezoelectric Materials and Applications in Actuators (IWPMA2010). 10-13 October, Antalya, Turkey, 2010.
5. Krishna, B.T. and Reddy, K. V. V. S. "Active and passive realization of fractance device of order ½". Active and Passive Electronic Components, 2008, 1-5.
6. Ozyetkin, M. M., Yeroglu, C., Tan, N., Tagluk, M. E. "Design of PI and PID controllers for fractional order time delay systems". 9th IFAC Workshop on Time Delay Systems (IFAC TDS 2010), June 7-9, Prague, Czech Republic, 2010.75