Optimal Control of a Smart Beam by using a Luenberger Observer

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ABSTRACT

This paper presents the design of an optimal vibration control mechanism, namely an LQR controller, with a Luenberger observer for a smart beam having surface bonded piezoelectric sensors and actuators. The approach intends to suppress the vibrations of the first flexural resonance of the smart beam. The smart beam studied was a cantilever aluminium beam with eight surface bonded Lead-Zirconate-Titanate (PZT) patches in bimorph configuration. The smart beam was excited at its first resonance frequency (approx. at 7 Hz) with a group of piezoelectric actuator patches and the response of the smart beam was monitored from a single piezoelectric sensor patch in order to obtain the necessary experimental frequency response for the system identification. The design of the controller was achieved by combining the optimal control law, architecture of Luenberger observer and inverse dynamic model of the smart beam. The verification of the developed controller was proved through the time and frequency domain responses and it was successfully shown that the intended target was achieved.

Keywords: *smart beam, PZT, Luenberger observer, vibration control*

1 Introduction

A smart structure usually consists of a passive metallic and or composite structure with elements called the smart materials. The smart structure can sense external disturbances and respond to those in real time. The smart materials are used as sensors and/or actuators and are either embedded in and/ or surface bonded to the existing passive structure [1, 2].

In this study, an active vibration controller was designed by using a Luenberger observer. The developed controller was applied on a smart aluminium beam in order to suppress the first flexural resonance vibrational levels. The Luenberger observer system was utilized for the estimation of the states of the problem. The smart beam values which were necessary for the estimation of the states of the problem were obtained from the inverse dynamics of the experimentally determined smart beam model. The controller was designed by combining the optimal control law, architecture of the Luenberger observer and inverse dynamic model of the smart beam. In order to validate the efficiency of the developed controller, time and frequency domain analyses were performed.

2 Smart Beam

The experimental studies were conducted on a smart beam. The smart beam has eight surface bonded SensorTech - BM500 (25 x 20 x 0.5 mm) PZT (Lead - Zirconate -Titanate) piezoelectric patches in bimorph condition [3]. The beam, which is widely used in the studies from METU, Aerospace Engineering Department [4, 6], is shown in Figure 1.

Figure 1: The smart beam used in the study

3 Luenberger Observer

In the theoretical studies, the optimal design is usually achieved by assuming that all states of the system are actually measurable. However, in real physical systems all states may not be measurable and hence some states need to be estimated. For the application of the optimal control usually an observer is used for the estimation of the states and this estimation is generally based on available limited sensing. Luenberger observer is known to be a state estimator with simple architecture and efficiency [7]. The architecture of the Luenberger observer is given in Figure 2.

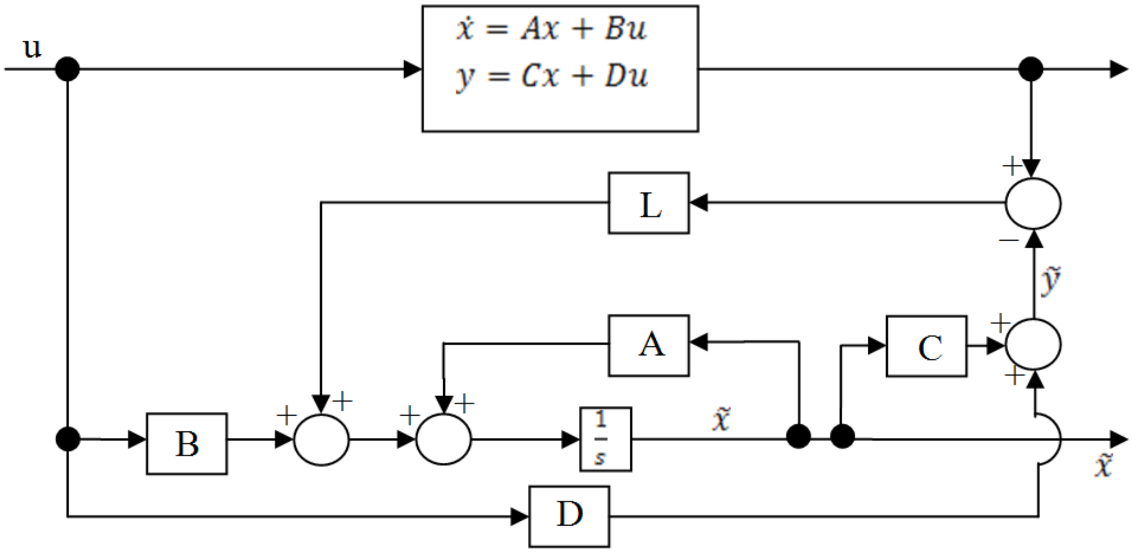

Figure 2: The architecture of Luenberger observer

where x defines the state vector and y stands for measurement vector. Matrices A, B, C and D are the state-space representation matrices for the system model.

The dynamics of the observer can be stated as:

$$\dot{\tilde{X}} = A\tilde{x} + Bu + L(y - \tilde{y}) \tag{1}$$

where $\tilde{x}$ is the estimated state vector and $\tilde{y}$ is the estimated measurement vector which are estimated by the observer.

The predictor part $(A\tilde{x} + Bu)$ of the above equation gives the plant dynamics. However, due to the possible uncertainties in the plant model, an estimate of the state based only on the predictor will be insufficient to represent the actual state of the system. Hence a correction term of $(L(y - \tilde{y}))$ is necessary and therefore the Luenberger observer is achieved. The added correction term is utilized in order to correct the future estimates based on the present existing error in the system. The observer gain L weighs the correction term in state estimation. It can be understood that a low value of L is chosen if the predictor is high and/or the measurements are noisy and a high value of L will be chosen for vice-versa.

4 Optimal Controller Gains

An optimal control approach is a full state feedback control problem. The performance index is calculated by considering all the state variables as [8],

$$J = \frac{1}{2}\int_0^\infty [x^T Qx + u^T Ru]dt \tag{2}$$

where x defines the state vector, Q is the quadratic measurement matrix, u is the input and R stands for the regulator gain. All the elements of the Q matrix and R are given as positive.

For an LQR controller, the optimum input u, is known to be [8]

$$u = R^{-1}B^{T}Px \tag{3}$$

where B is the state space representation matrix described in section 3 and P is the solution of the Riccati equation given below.

$$A^{T}P - PBR^{-1}B^{T}P + PA + Q = 0 \tag{4}$$

In Equation (4) A is one of the state space representation matrices defined in section 3. In this study R is taken as 0.01 and the Q is determined as,

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{5}$$

5 Controller Design

In order to suppress the vibrations of the smart beam the following values are used. G_p is the experimentally obtained system model of the smart beam and G_p^{-1} is its inverse dynamics.

$$G_p = \frac{0.068s^2 + 0.076s + 146.1}{s^2 + 0.273s + 1933} \tag{6}$$

$$G_p^{-1} = \frac{s^2 + 0.273s + 1933}{0.068s^2 + 0.076s + 146.1} \tag{7}$$

A_p, B_p, C_p ve D_p are the experimentally obtained state space matrices of the smart beam model which are given in Equations (8) to (11).

$$A_p = \begin{bmatrix} -0.16 & -43.96 \\ 43.96 & -0.11 \end{bmatrix} \tag{8}$$

$$B_p = \begin{bmatrix} 0.43 \\ -0.36 \end{bmatrix} \tag{9}$$

$$C_p = \begin{bmatrix} 0.43 & 0.36 \end{bmatrix} \tag{10}$$

$$D_p = [0.068] \tag{11}$$

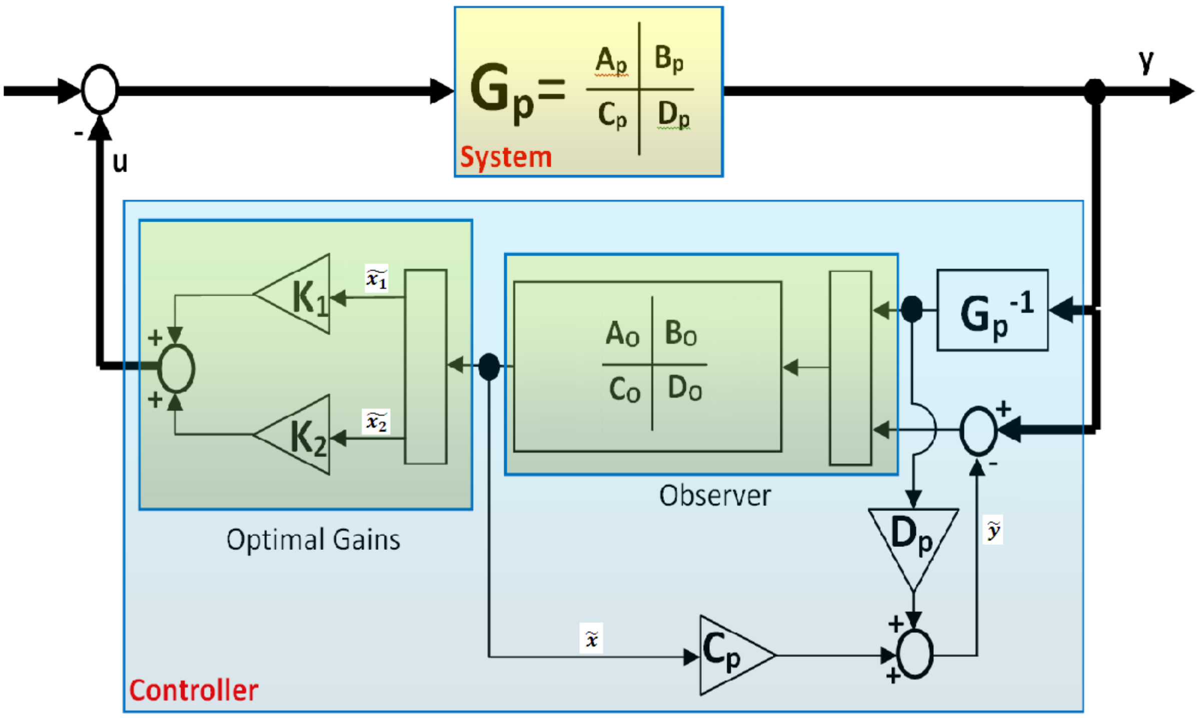

Figure 3: Block diagram of the developed optimal controller

where A_O, B_O, C_O ve D_O are the state space matrices of the observer and are given in Equations (12) to (15). The design of the observer was completed according to the observer gain vector L given in Equation (16). K_1 and K_2 are the optimal control gains and in the current study are calculated as K_1=10.8573 and K_2= -8.3020

$$A_o = \begin{bmatrix} -0.16 & -43.96 \\ 43.96 & -0.11 \end{bmatrix} \tag{12}$$

$$B_o = \begin{bmatrix} 0.43 & 9255 \\ -0.36 & -8332 \end{bmatrix} \tag{13}$$

$$C_o = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{14}$$

$$D_o = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{15}$$

$$L = \begin{bmatrix} 9255 \\ -8332 \end{bmatrix} \tag{16}$$

6 Experimental Results of the Smart Beam

In order to investigate the effectiveness of the designed controller in the frequency domain, the smart beam was excited through a sweep sine from 2 Hz to 18 Hz by covering the first resonance frequency which is around 7 Hz. The experimentally obtained open and closed loop frequency response curves of the smart beam are given in Figure 4.

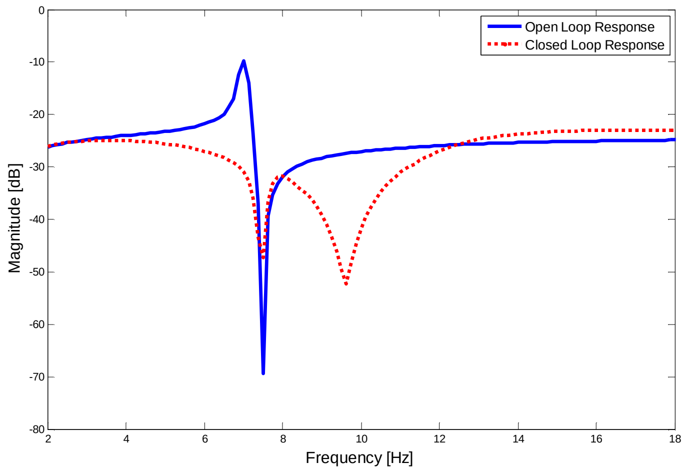

Figure 4: Experimental frequency responses of the smart beam

The smart beam was then given an initial 8 mm tip deflection and the ensuing motion was measured for open and closed loop time responses which are presented in Figure 5. It can easily be observed from the figure that the settling time is around 50 seconds for the open loop behaviour. On the other hand, the designed controller managed to suppress the free vibrations of the smart beam with a settling time of nearly 6 seconds. Then, the smart beam was also excited at its first resonance frequency (around 7 Hz.) by the help of PZT patches. The efficiency of controllers on the suppression of the forced vibrations are also shown in Figure 6.

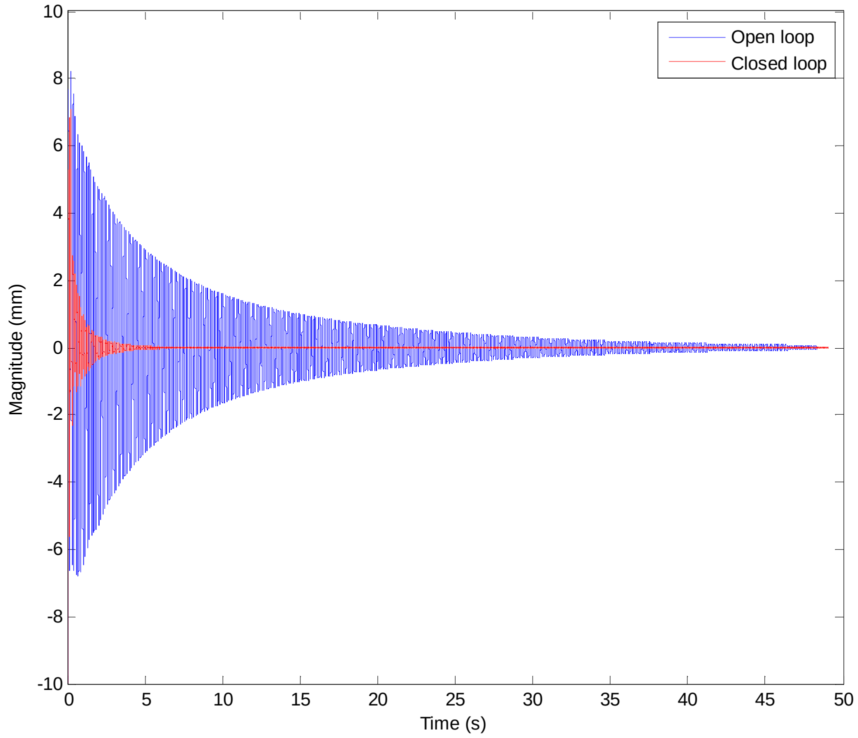

Figure 5: Experimental free vibrations, time responses of the smart beam

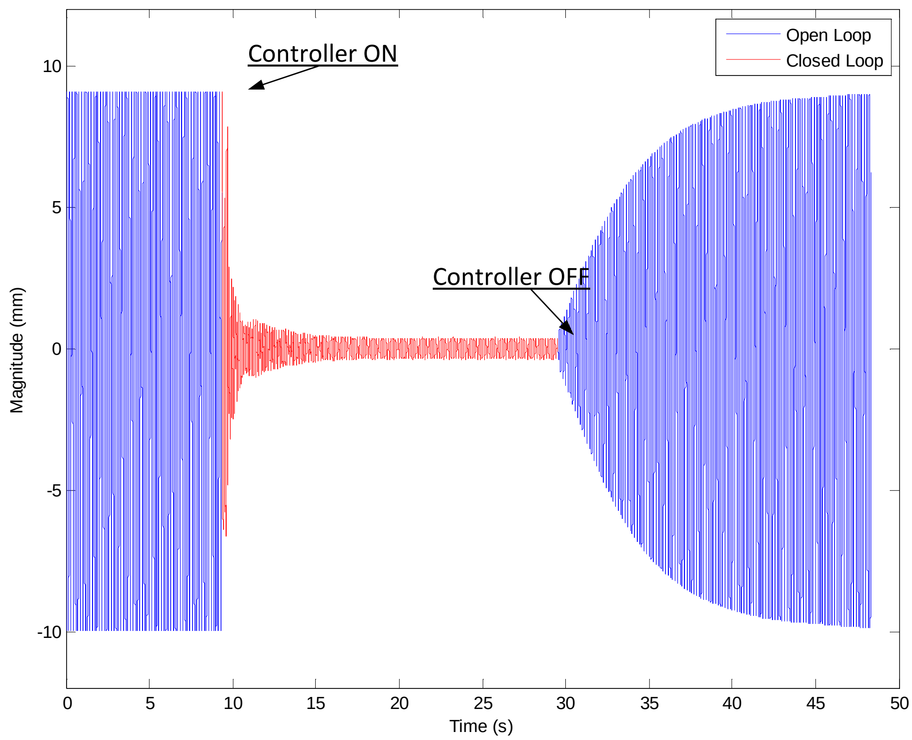

Figure 6: Experimental forced vibrations, time responses of the smart beam at its first resonance frequency

7 Conclusions

In this study, the design and implementation of an optimal vibration control mechanism, so-called an LQR controller with a Luenberger observer, was considered in order to suppress the first flexural vibrations of a smart beam. The experimental results, which were obtained both in time and frequency domain, show that the designed controller was capable of successfully reducing the vibration levels.

References

1. Caliskan T (2002). *Smart Materials and Their Applications in Aerospace Structures*. PhD Thesis, Middle East Technical University.
2. Ulker D F (2003). *Active Vibration Control of Smart Structures*. Master Thesis, Middle East Technical University.
3. Sensor Technologies Limited (2002). BM-500 Lead Zirconate Titanate Product Data Sheet.
4. Sahin M Karadal F M Yaman Y Kircali O F Nalbantoglu V Ulker F D Caliskan T (2008). Smart Structures and Their Applications on Active Vibration Control: Studies in the Department of Aerospace Engineering, METU. *J. Electroceramics*, 20(3-4), 167–174.
5. Kircali O F Yaman Y Nalbantoglu V Sahin M Karadal F M Ulker F D (2008). Spatial Control of a Smart Beam. *J. Electroceramics*, 20, 175–185.
6. Onat C Sahin M Yaman Y (2011). Active vibration suppression of a smart beam by using an LQG control algorithm. *2nd International Conference of Engineering Against Fracture (ICEAF II)* 22-24 June, Mykonos, Greece.
7. George E (2012). *Control System Design Guide*, Kollmorgen Corporation, Chapter 10. Pages 185-212.
8. Anderson B D O Moore J B (1990). *Optimal Control: Linear Quadratic Methods.* Prentice-Hall, N.J., 1990.