Decomposition Methods in MDO
Decomposition Methods in Design

• Decomposition strategies have been used in the design of multidisciplinary systems
  – hierarchic decomposition
  – non-hierarchic decomposition
  – hybrid decomposition

• Elements of decomposition
  – partitioning
  – co-ordination
Decomposition Methods in Design

Hierarchic

Airframe
- Wing
- Fuselage
- Torque Box
- Spar

Non-Hierarchic

Aerodynamics
- Structures
- Controls

Hybrid

Aerodynamics
- Structures
- Controls
- Torque Box
- Spar
In general, the minimal weight corresponding to $A_2=0$ will never be achieved.
Sequential Approach

- Aerodynamics
- Structural Design
- Aeroelastic Design
- Control Systems

Each discipline may contain inner loops of iteration.

Initial Concept

Interdisciplinary Iterations
Hierarchic Decomposition Based Design

- **System level**
  - P - problem parameters
  - X - design variables
  - Q - behavior variables
  - Analysis $F(X,P,Q)=0$
    - Z passes as input to subsystem $Z=f(X,Q)$
- **Subsystem level**
  - x - local design variables
  - q - local behavior variables
  - analysis $f(Z,q,x)=0$

Diagram:

- SYSTEM
- Subsystem 1
- Subsystem 2
- $P \rightarrow X$
- $Z \rightarrow Z \leftarrow Z$
- $C_1^0 \leftarrow \frac{\partial C}{\partial X} \rightarrow C_2^0$
- No lateral interaction
Hierarchic Decomposition Based Design

- Subsystem level optimization
  - \( g = g(q(Z(X),x) \leq 0 \) and \( h = h(Z(X),x) = 0 \) are the inequality and equality constraints respectively
  - \( C \) is a cumulative constraint representation of all inequality constraints
- Optimization problem statement

\[
\text{Minimize } C \text{ to obtain } C_i^0 \\
\text{Subject to } h = 0
\]

- Pass back \( C_1^0, \frac{\partial C}{\partial X} \) and \( C_2^0, \frac{\partial C}{\partial X} \) to the system level
Hierarchic Decomposition Based Design

- System level optimization
  - $F$ is system objective $F = F(Q(X))$
  - $G$ are system constraints $G = G(Q(X))$
- Mathematical problem statement

$$\begin{align*}
\text{Minimize} & \quad F(X) \\
\text{Subject to} & \quad G_j \leq 0; \quad C_i \leq 0 \text{ for all } i \text{ subsystems}
\end{align*}$$

- Here $C_i$ are obtained as a linear extrapolation

$$C_i = C_i^0 + \frac{dC}{dX} \Delta X$$

$$\frac{dC}{dX} = \frac{\partial C}{\partial Z} \frac{\partial Z}{\partial X}$$

$\uparrow$ System sensitivity

$\uparrow$ Subsystem optimal sensitivity
Hierarchic Decomposition Based Design

- Errors in extrapolation due to active constraint switching
- Data management gets involved in realistic, large-scale MDO problems
- Handling of equality constraints is required as these provide system to subsystem coordination - proves to be problematic!
Non-Hierarchic Decomposition Based Design - CSSO

- Concurrent Subspace Optimization - Overview

Minimize \( f(x^k) \)

Subject to: \( C^p \leq C^{po} [s^p (1 - r_k^p) + (1 - s^p) t_k^p] \quad p = 1, nss \)

\[ x^k_L \leq x^k \leq x^k_U \]

- \( C^p \) is a measure of all constraints in subspace \( p \), super and subscript \( p \) and \( k \) denote the influence of subspace \( p \) on subspace \( k \), \( r \)'s are the responsibility coefficients, \( t \)'s are the trade-off coefficients, and \( s \) are the switch parameters
CSSO Overview

- Optimum in each subspace is a function of r and t coefficients, and a second level problem needs to be solved - COP

\[
\begin{align*}
\text{Minimize} & \quad F = f^0 + \sum_p \sum_k \frac{df}{dr_k} \Delta r_k^p + \sum_p \sum_k \frac{df}{dt_k} \Delta t_k^p \\
\text{Subject to:} & \quad \sum_k r_k^p = 1 \quad p, k = 1, nss \\
& \quad \sum_k t_k^p = 0 \quad 0 \leq r_k^p \leq 1 \\
& \quad r_k^p \leq r_k^p \leq r_k^p \\
& \quad t_k^p \leq t_k^p \leq t_k^p
\end{align*}
\]

- COP yields a new set of r’s and t’s to be used in next round of SSO’s
Deficiencies in CSSO

- Coupling in CSSO is resolved through the use of linear or higher-order approximations. These require move limits to be placed on the design variables.
- The formulation of the coordination problem is based on an optimal sensitivity analysis procedure that cannot be regarded as robust.
- The use of heuristics in solving the coordination problem have proven to be of limited benefit, often introducing cycling and convergence problems.
- Sensitivity information is unavailable when dealing with discrete and integer design variables.
- Solution of GSE for computing global sensitivity information is problematic in terms of the required computational effort, and numerical problems such as singularity and ill-conditioning.
Collaborative Optimization

 coupled

 uncoupled

 target state

 optimizer

 uncoupled
Collaborative Optimization - Example

$X_{F1}$ - cross sectional dimensions of stringers and skins

$X_M - Y_{21}$ and $X_{shared}$

$g(X)$ - constraints on stress and displacements

$X_{shared}$ - wing taper, wing sweep, wing aspect ratio

$X_{F2}$ - airfoil leading edge radius and camber

$X_M - Y_{12}$ and $X_{shared}$

$g(X)$ - constraints on pressure distribution gradients
Collaborative Optimization - Procedure

Optimization 1
Minimize $J(X_1)$
subject to $g(X_1) \leq 0$

Optimization 2
Minimize $J(X_2)$
subject to $g(X_2) \leq 0$

System Level Optimization

Target Vector
$Z = \begin{Bmatrix} X_{shared}^T \\ Y_{12}^T \\ Y_{21}^T \end{Bmatrix}$

$J = \begin{Bmatrix} Y_{21} - Y_{21T} \\ X_{shared} - X_{sharedT} \\ Y_{12} - Y_{12T} \end{Bmatrix}^T \begin{Bmatrix} Y_{21} - Y_{21T} \\ X_{shared} - X_{sharedT} \\ Y_{12} - Y_{12T} \end{Bmatrix}$

$J = \begin{Bmatrix} Y_{12} - Y_{12T} \\ X_{shared} - X_{sharedT} \\ Y_{21} - Y_{21T} \end{Bmatrix}^T \begin{Bmatrix} Y_{12} - Y_{12T} \\ X_{shared} - X_{sharedT} \\ Y_{21} - Y_{21T} \end{Bmatrix}$
Collaborative Optimization
System Level Optimization

- Find $Z$ (system level variables shared by different modules) to minimize $F(Z)$ (system level objective)
- Satisfy $J_i=0$ for all modules $I$
  - here $J_i$ is obtained from extrapolation based on optimal problem parameter sensitivity from each module

$$J_i = (J_i)^{old} + \left[ \frac{\partial J_i}{\partial Z} \right]^T \{\Delta Z\}$$
Collaborative Optimization Summary

- Each module optimized separately - can be done concurrently
- System level coordination optimization is generally with small number of variables
- Each discipline is allowed to function autonomously
- There is no explicit system analysis - instead we have a situation akin to simultaneous analysis and design